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THE

ARITHMETIC OF DECIMALS,

ADAPTED TO A

DECIMAL COINAGE.

BY

JAMES BRYCE, M.A., LL.D.

Second Edition, Enlarged.

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THE

ARITHMETIC OF DECIMALS

ADAPTED TO A

DECIMAL COINAGE

WITH COPIOUS ILLUSTRATIONS OF IMPROVED MODES
OF RECKONING

BY

JAMES BRYCE, M.A., LL.D.
FELLOW OF THE GEOLOGICAL SOCIETY

SECOND EDITION, ENLARGED

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PREFACE.

THE plan of a Decimal Coinage, recommended by a Select Committee of the House of Commons, in a Report dated the 1st of August, 1853, was received with very general acceptance throughout the country. In advocacy or illustration of it several useful tracts, and an able and learned work by Sir John Bowring, soon after appeared. All of these, so far as the Author is aware, were intended merely to supplement the ordinary treatises on Arithmetic; they did not profess to explain the general theory of decimal fractions, or the reasons of the rules for the management of these quantities. It appeared to him, however, that such an exposition, in connection with the application of decimals to mercantile calculations, in which fractions constantly occur, and with an enforcement and illustration of the plan of the Committee, would be found useful by those who had not given much attention to the subject of Decimals. Besides, many treatises on Arithmetic, even of recent date, and excellent in other respects, contain no notice whatever of the Contracted Methods of multiplying and dividing decimals; yet much of the benefit which will arise from the adoption of a decimal coinage, will depend upon the employment of these methods, as was long ago pointed out by Professor De Morgan, when advocating a plan almost identical with that since recommended by the Committee. Hence it has been thought best to give a connected view of the whole subject of Decimals, setting forth as simply as possible the reason of every rule, with a full explanation of the Contracted Methods above referred to, and copious exercises in illustration of the various processes. The applications are shown separately in a second Part, as well with our present denominations of money, as with those of the proposed coinage. It is hoped that these applications will be found especially useful in directing attention to improved methods of calculating, even irrespective of a decimal coinage. To shorten the calculations as much as possible, several tables, applicable to both coinages, have been appended. By the help of two of these, a person who can multiply with tolerable rapidity, and is expert in the use of the abbreviated methods, will work any question in Interest or Discount more quickly than by "the present mechanical reference to tables," such as are used in our Banks, and be more certain of his result.

It will become highly desirable, indeed necessary, should the decimal system be adopted, to introduce the learner to a knowledge of decimals at a much earlier period of his course than is now attempted. But this might very easily be done by presenting them as originating in a simple extension of the Arabic notation to the right of the unit's place, as explained in Arts. 1 and 2. And as fractions are first met with in Simple Division, so much might then be easily taught regarding them as would render the elementary principles of decimal fractions perfectly intelligible to young arithmeticians. The theory of measures and multiples, and the management of fractions generally, could be taken up at an after period. But, as is shown in these pages, the decimal, like our present denominations of money, may be presented to the pupil's mind without reference to fractions; and he could thus, or in the way above indicated, pass at once from the "Elementary Rules," to the applications of the decimal system in the rules of Proportion and Practice. The grand object to be gained is to sweep away "Reduction" and the "Compound Rules" from our arithmetical books. The introduction of a decimal coinage would do this, so far as money is concerned;—the adoption of a well devised metrical system, on a decimal plan, would destroy the whole fabric, and remove a most serious hindrance from the learner's path; he would gain at a single bound a high vantage-ground, from which his upward progress would be rapid and decided. Time would then be found for carrying pupils farther than the great majority can now go,—into the higher arithmetic, algebra, and mensuration; and for the cultivation of other branches, eminently fitted to promote the growth of the faculties, and interest the minds of young persons.

The benefits, however, would not be confined to education; they would extend to every department of commercial calculation, and to the keeping of all sorts of accounts, both in public and private establishments. But these have been so often and so well set forth, that they need not be here enumerated. It is earnestly to be desired that a system "so complicated as the present, and so utterly wasteful" of precious time, should be speedily abolished.—The attempt lately made by merchants in Liverpool and Glasgow to introduce the cental as a substitute for the hundredweight in weighing grain, though only partially successful, is a hopeful symptom of the increasing favour with which the plan of decimalizing our weights and measures, as well as our money, is viewed by the most intelligent and best educated portion of the commercial community.

January 6, 1860.

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THE

ARITHMETIC OF DECIMALS.

ART. 1. About the year 1580, Simon Stevinus of Bruges conceived the happy idea of extending the figures in the common system of notation downwards from the unit's place. This simple contrivance completed the Arabic system; and its beautiful and perfect "device of place," by which we are enabled readily to express all numbers to any extent above unity, was rendered equally applicable to the expression of fractions, that is, numbers less than unity.[1]

2. Since in the Arabic system the digits decrease towards the right, so that the value of each is the *tenth part* of that which it would have if it occupied the preceding place, it is obvious that, if we take the number 7777, and, marking the unit's place by a bar, annex three other figures to the right of that place,—thus,

7777 | 777,

the first figure to the right of the bar will denote a number which will be the tenth part of 7 units, that is $\frac{7}{10}$ (one-tenth of 7, or seven-tenths of 1); the second figure to the right of the bar will denote a number one-tenth of this $\frac{7}{10}$, that is $\frac{7}{100}$; the third figure will denote $\frac{7}{1000}$, and so on. So also, in any other number, as

---

[1] The invention of decimal fractions has been ascribed to John Müller, who flourished at Königsberg ("Kings-Mount"—whence he was called Regio-Montanus, after the fashion of the times), between the years 1474-1533. Müller's invention, however, though important, was of a different character—he was the first to suggest, in his *Tracts on Trigonometry*, a decimal division of the radius. Stevin does not state that his invention completes the Arabic-system; and Professor De Morgan even thinks it probable that it did not occur to him that it completed it. The invention was published in 1585, in a tract called the *Disme*. (See a masterly paper by De Morgan in the *British Almanack and Companion for* 1851; *Arithmetical Books*, by the same author; and Peacock's History of Arithmetic, in the *Encyclopædia Metropolitana*.)

$$1462 \mid 375,$$

the first figure to the left of the bar denoting units, the first figure to the right of the bar will denote tenths of units; the second tenths of this, or hundredths; the third thousandths, and so on. The fractional part is therefore

$$\tfrac{3}{10}, \tfrac{7}{100}, \text{ and } \tfrac{5}{1000};$$

and since the sum of these fractions is

$$\tfrac{375}{1000},$$

the three numbers may be taken together, and the fraction named 375 thousandths. Instead of the separating bar, or one placed below, thus,

$$1462\underline{375},$$

both of which were used at first, a dot or full point has been long employed. This dot, called the *decimal point*, is placed in front of the upper part of the first figure, thus, 1462·375.[2]

Thus also ·03 denotes $\frac{0}{10}$ (no tenths) and $\frac{3}{100}$; ·007, $\frac{0}{10}$, $\frac{0}{100}$, and $\frac{7}{1000}$, or simply $\frac{7}{1000}$; ·0004, $\frac{4}{10000}$, &c.

3. It follows, from the nature of this notation, that in suppressing the denominator of a fraction, and writing the numerator with the decimal point before it, the last figure of this numerator must occupy the place denoted by the number of zeros in the denominator, hundredths

---

[2] It is placed in front of the upper part of the figure to distinguish it from one of the marks used to denote multiplication, which is placed in front of the lower part of the multiplier; thus, $5 . 6 = 30$, *read* 5 into 6 equals 30. The other mark denoting multiplication is × (*into*); thus $4 \times 6 = 24$. When letters are employed to denote numbers, their multiplication is indicated by writing them close together like the letters of a word. Thus ax denotes the product of the factors a and x; or if $a = 7$ and $x = 5$, $ax = 35$. Also, $10x$ and $8y$ denote ten *times* x and eight *times* y. The sign + (*plus*), indicates addition; thus, $4 + 6 = 10$; the sign — (*minus*), subtraction, as $11 - 7 = 4$. Division is indicated by the marks ÷ or :, as well as by writing the dividend above the divisor and drawing a line between them; thus, $21 \div 7$, $21 : 7$, and $\frac{21}{7}$, all denote division, and the quotient is 3.

in the second place, thousandths in the third place, ten-thousandths in the fourth, and so on. Thus, if we express the fraction $\frac{6}{10000}$ without denominator, the figure 6 must be thrown back into the fourth place by prefixing ciphers, that being the place of ten thousandths, so that it is written ·0006. Also, $\frac{32}{1000}$ is written ·032; the number of places of figures after the decimal point being always the same as the number of ciphers in the denominator.

4. Inasmuch as the successive powers of 10 consist only of the number 1, followed by one or more ciphers,[3] a decimal fraction may be defined to be *a fraction whose denominator is any power of* 10. And since the number of ciphers in any power is the same as the number of units in the index of that power, the number of figures after the decimal point will also be the same as the index of that power of 10 which is equal to the denominator.

5. It follows also from the nature of this notation that, if

---

[3] *a.* These powers are, *second* 100, *third* 1,000, *fourth* 10,000, &c. A *power* of a number is the product of two or more factors, each equal to that number. The powers are named from the number of factors; and the number itself, for the sake of uniformity, is called the *first* power. The second and third powers are also called, respectively, the *square* and *cube*. Thus,

The 1st power of 3 is 3,
2d ... 3 ... 9,
3d ... 3 ...27,
4th ... 3 ...81, &c.

b. In order to indicate how many factors a power contains, a number is written over it, and towards the right hand; and the number so placed is called the *index* or *exponent* of the power. Thus $4^3=64$, and 3 is the index. The successive powers of 10 will therefore be written as follows:—

$10^0=1$
$10^1=10$
$10^2=100$
$10^3=1,000$
$10^4=10,000$
$10^5=100,000$, &c.

The number of ciphers in each of these powers is plainly the same as the number of units in the index of that power; *two* ciphers in the second power, *three* in the third power, &c. Also, from the nature of simple multiplication, it is obvious that, when two or more of these powers are multiplied together, the product will contain as many ciphers as there are in all the factors. Thus, the product of 100 by 1000, or of 10^2 by 10^3, contains 5 ciphers; or $100 \times 1000 = 10^2 \times 10^3 = 10^{2+3} = 10^5 = 100,000$; and $10^2 \times 10^3 \times 10^4 = 10^{2+3+4} = 10^9 =$ 1,000,000,000, which contains $2+3+4$, that is, 9 ciphers.

the decimal point be removed one place towards the right, the value of every figure is increased tenfold, that is, the number is multiplied by 10; and if it be removed one place towards the left, the value of each figure is one-tenth of its former value, or the number is divided by 10. So also, to multiply or divide by 100, 1000, &c., we have only to remove the point two, three, &c., places toward the right or left. Thus, the number

56·4375

is multiplied by 1000 by placing the point between 7 and 5, and writing

56437·5,

and the number 124·728

is divided by 100 when we write

1·24728.

6. It is also manifest that ciphers at the end of a decimal do not change the value of the figures in that decimal: since, for every cipher in such a position a cipher may be cut off from the denominator: and that, therefore, they may be omitted when they occur, or may be affixed when required in any operation. Thus, $\cdot 600 = \frac{600}{1000} = \frac{60}{100} = \frac{6}{10} = \cdot 6$.

EXERCISES.

1. Express ·003 with denominator.
2. — ·0027 —
3. — $\frac{7}{100}$ without denominator.
4. — $\frac{371}{100000}$ —
5. — 5 tenths + 7 thousandths decimally.
6. — 3 hundredths + 11 ten-thousandths decimally.
7. — 9 thousandths + 27 millionths decimally.
8. Multiply 154·736 by 100.
9. — 18·425 by 10,000.
10. — ·0011 by 1,000.
11. — ·5796 by 100.
12. Divide ·2 by 10.
13. — 12·875 by 100.
14. — ·01 by 1,000.
15. — 8756·625 by 1,000.

REDUCTION.

7. The structure of the denominators, as above explained, suggests a ready mode of *reducing* any fraction *to a decimal*, that is, of expressing the fraction by an equivalent decimal. Such an operation must be performed as shall give 1 for the first figure of the denominator, and for the other figures, ciphers. Annex, therefore, the same number of ciphers to both terms (in order to preserve the equivalence), and divide both terms by the denominator of the given fraction; the first figure of the new denominator will then be 1, and the other figures ciphers; and the fraction will be a decimal. Thus, let the fraction be $\frac{15}{16}$; annexing four ciphers to each term, that is, multiplying both terms by 10,000, we have

$$\frac{150000}{160000}.$$

Dividing both terms by 16, in order to obtain 1 for the first figure of the denominator, we find

$$\frac{9375}{10000} \text{ or } \cdot 9375,$$

for the equivalent decimal required.

Here, and in many other cases, it is easy to see how many ciphers are needed; but, as it generally happens that this is not apparent, it is better to annex a cipher at each step of the division, which is carried on *till the numerator is obtained;* the denominator can then be at once supplied, by the principle of Art. 3. The above process will therefore take the more convenient form, indicated on the margin.

$$\begin{array}{rl} 16\,)\,150 & (\,\cdot 9375 \\ \underline{144} & \\ 60 & \\ \underline{48} & \\ 120 & \\ \underline{112} & \\ 80 & \\ \underline{80} & \\ 0 & \end{array}$$

The operation here terminates with the fourth figure of the quotient, the divisor being contained exactly in the dividend, after the fourth cipher has been annexed to the remainder 8. But remainders will often arise, giving, when ciphers are annexed, dividends in which the divisor is *not* contained *exactly*, so that the quotient will never terminate. Thus, in reducing $\frac{1}{3}$ to a decimal, the same remainder continually recurs, and the dividend is always 10, so that the quotient will never terminate. Or it may happen that the same remainder will recur after *several* steps in the division, so that the figures in the quotient will also recur continually. The two examples which follow, the reduction of $\frac{7}{12}$ and $\frac{5}{13}$ to decimals, illustrate this:—

$$\begin{array}{l}
3)\,10\,(\cdot333\ldots. \\
\quad\;\; 9 \\
\quad\;\; \overline{10} \\
\quad\;\;\;\; 9 \\
\quad\;\;\;\; \overline{10} \\
\quad\;\;\;\;\;\; 9 \\
\quad\;\;\;\;\;\; \overline{1},\ \&c.
\end{array}$$

$$\begin{array}{l}
12)\,70\,(\cdot58333\ldots \\
\quad\;\; 60 \\
\quad\;\; \overline{100} \\
\quad\;\;\;\; 96 \\
\quad\;\;\;\; \overline{40} \\
\quad\;\;\;\; 36 \\
\quad\;\;\;\;\;\; \overline{40} \\
\quad\;\;\;\;\;\; 36 \\
\quad\;\;\;\;\;\;\;\; \overline{40} \\
\quad\;\;\;\;\;\;\;\; 36 \\
\quad\;\;\;\;\;\;\;\;\;\; \overline{4},\ \&c.
\end{array}
\qquad
\begin{array}{l}
13)\,50\,(\cdot384615384615\ldots \\
\quad\;\; 39 \\
\quad\;\; \overline{110} \\
\quad\;\; 104 \\
\quad\;\;\;\; \overline{60} \\
\quad\;\;\;\; 52 \\
\quad\;\;\;\; \overline{80} \\
\quad\;\;\;\; 78 \\
\quad\;\;\;\;\;\; \overline{20} \\
\quad\;\;\;\;\;\; 13 \\
\quad\;\;\;\;\;\; \overline{70} \\
\quad\;\;\;\;\;\; 65 \\
\quad\;\;\;\;\;\;\;\; \overline{5},\ \&c.
\end{array}$$

In the first, after the second step, the remainder is always 4, and the figure 3 continually recurs in the quotient. In the second, after the sixth step in the process, the first dividend 50 recurs, and the figures of the quotient, recurring in the same order, will be repeated without end.

8. There are thus two classes of decimals: the *finite* or *terminate*, and the *interminate* or *recurring*. To the latter belong much the larger number of decimals which occur in computations;—the greater number of fractions are not expressible decimally in finite terms. We may, however, approximate as nearly as we please to the decimal equivalents of such fractions by continuing the quotient; and since the values of the successive figures decrease so rapidly (tenths, hundredths, thousandths, &c.), even a small number of decimal places suffices for this purpose. Thus, in the last example, if we reject all the decimals after the

sixth place, the error will be less than one-millionth part of one of the units contained in the numerator 5; for a unit in any place, whether of decimals or whole numbers, by the nature of our common notation, is greater than the sum of all the figures which follow it, how great soever these be; so that the omitted part of the last quotient (·384615384615...), even if the sixth figure were 1 and the following figures all 9s, would differ from the true decimal equivalent of $\frac{5}{13}$ by less than one-millionth of one of the units contained in the numerator 5. Hence no appreciable error can be introduced by using these approximate values; it is only necessary to carry out the quotients to a greater number of places in the case of a large unit of measure, such as 1 ton, 1 mile, &c.

9. A fraction before being reduced to a decimal is expressed in its lowest terms, so that the numerator and denominator have no factors in common; ciphers are then affixed to the numerator, that is, it is multiplied by 10, 100, 1000, or other power of 10. Hence, in order that the decimal shall terminate, *these powers* must be divisible by the denominator. Now, the only factors of 10 are 2 and 5; and of the powers of 10, these two numbers and their powers; and, therefore, unless the denominator be composed of either, or both, of these factors, or of powers of them, the decimal will never terminate, that is, an *exact* decimal equivalent of the given fraction can never be found. Fractions, then, which are expressible in finite decimals, are only such as have for their denominators 2, 4, 8, 16, 32, 64, 128, &c.; 5, 25, 125, 625, &c., or the products of these numbers, as 20, 40, 50, 250, &c.; but if 3, 7, 11, &c., or their powers, or any prime number except 2 or 5, enter the denominator as factors, the decimal will be interminate.

10. Now that all *interminate* decimals arising from such divisions must consist of figures which *recur* in some particular order is manifest; for, since no remainder can occur as great as the divisor, that is, the denominator of the given fraction, some remainder already found must recur before we have obtained as many remainders as there are

units in the divisor; and since a cipher is always annexed, the same dividend will always recur, and therefore the figures of the quotient will come round in the same order. Thus, in reducing $\frac{6}{7}$ to a decimal, after all the possible remainders, 1, 2, 3, 4, 5, 6, have occurred in the order 6, 4, 5, 1, 3, 2, the remainder 6 again occurs, and the quotient is ·857142857142, &c., without end. Also, in reducing $\frac{7}{19}$ to a decimal, after 18 different remainders, the greatest number possible, have occurred, the figure 7, the first remainder, again occurs, and the quotient figures are also repeated. In reducing $\frac{16}{17}$ to a decimal, it will be found that the same remainder does not recur till 16 places in the quotient are obtained—and fractions in their lowest terms, which have for denominators 19, 23, 29, 47, 59, 61, 97, 109, 113, 131, 149, 167, 181, 193, &c., do not give recurring figures till the greatest possible number of remainders (one less than the number of units in each denominator) has been obtained. In general, however, it is only by trial that the extent of the period can be ascertained. It is easy to see, from the reasoning above, that if a decimal follow no such law of recurrence, but is continued *indefinitely*, it cannot be generated from any fraction expressible in common finite terms.

11. These recurring decimals are also called *circulating* or *periodic;* and the figures which repeat are called a *period.* The first and last figures of the period are marked by a trait (′) or dot (˙). Thus, in the decimal equivalent of $\frac{5}{13}$, the period consists of 6 figures, and is written $\cdot 3'84615'$ or $\cdot\dot{3}8461\dot{5}$;

the equivalent of $\frac{1}{3}$, or ·333 &c., is written

$$\cdot 3' \text{ or } \cdot\dot{3},$$

the period consisting of a single figure. Such decimals are sometimes, and perhaps more properly, termed *repeaters.* *Pure circulating* decimals are those which repeat from the beginning, as $\cdot\dot{3}$, $\cdot\dot{0}\dot{4}$, $\cdot\dot{3}8461\dot{5}$; those which begin to repeat after a certain number of figures, are called *mixed circulating* decimals, as $\cdot 58\dot{3}$, $\cdot 94\dot{7}2\dot{4}$.

EXERCISES.

Reduce the following fractions to decimals :—

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{12}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{8}$ $\frac{7}{8}$, $\frac{10}{11}$, $\frac{11}{12}$.
3. $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$. 4. $\frac{5}{64}$.
5. $\frac{5}{1024}$. 6. $\frac{1}{128}$. 7. $\frac{11}{32}$. 8. $\frac{15}{256}$. 9. $\frac{71}{1024}$.
10. $\frac{199}{200}$. 11. $\frac{79}{80}$. 12. $\frac{24}{25}$. 13. $\frac{123}{125}$. 14. $\frac{249}{250}$.
15. $\frac{157}{625}$. 16. $\frac{1}{4}$ of $\frac{2}{3}$ of $\dfrac{3}{3\frac{1}{5}}$. 17. $\frac{3}{7}$ of $1\frac{2}{19}$ of $1\frac{1}{75}$.
18. $\dfrac{47\frac{5}{8}}{94}$ of $\dfrac{11\frac{3}{4}}{7\frac{1}{2}}$. 19. $\frac{1}{81}$. 20. $\frac{14}{15}$. 21. $\frac{1}{99}$, $\frac{1}{999}$, $\frac{1}{9999}$.
22. $\frac{7}{53}$. 23. $\frac{6}{10001}$. 24. $\frac{16}{17}$. 25. $\frac{18}{19}$. 26. $\frac{37}{79}$.
27. $\frac{4111}{33300}$. 28. $\dfrac{2\frac{1}{2}\times 2\frac{1}{2}\times 2\frac{1}{2}-1}{2\frac{1}{2}\times 2\frac{1}{2}-1}$. 29. $\frac{1}{13}$, $\frac{1}{14}$. 30. $\frac{1}{49}$.

12. *To reduce a given quantity to the decimal of another* of the same kind, regarded as an integer :—Reduce both to the same denomination, express the one as a fraction of the other, and reduce this fraction to a decimal.

Ex. 1. Reduce 15s. 10½d. to the decimal of £1.

Here 15s. 10½d. and £1 being reduced to halfpence, and the less divided by the greater, we have the fraction $\frac{381}{480}$. The simplest form of which is $\frac{127}{160}$; and this being reduced in the ordinary way (Art. 8), gives the decimal

$$\cdot 79375 \text{ of } £1,$$

or as it is written,

$$£\cdot 79375.$$

In many cases these reductions may be more readily effected. Thus, in the above example, since 10½d. $=\frac{7}{8}$s. $=$ s·875, by expressing both as shillings, we have for the required fraction

$$\frac{15\cdot 875}{20}=\frac{1\cdot 5875}{2}=\cdot 79375.$$

Ex. 2. Reduce 13 cwt. to the decimal of 1 ton.

The fraction is here $\frac{13}{20}$; multiplying both terms by 5 we find $\frac{65}{100}=\cdot 65$ for the decimal required.

Ex. 3. Reduce $\frac{15}{16}$ cwt. to the decimal of a ton.

Since $\frac{15}{16}$ = ·9375, dividing by 20 (10 and 2) we obtain at once ·046875 for the required decimal.

Ex. 4. Reduce 12s. 6¾d. to the decimal of £10.

This question may be more readily worked as on the margin, than by reducing both to farthings, and the resulting fraction to a decimal.

The same method may be often more conveniently applied in the case of compound quantities, than that stated in the above Rule, as shown in the following example.

6¾ = 6·75

12	6·75
20	12·5625
10	·628125
	£·0628125

Ex. 5. Reduce 12 cwt., 1 qr., 23 lbs., 10 oz., 14 drs., to the decimal of a ton.

Reducing the given quantity and 1 ton to drams, we find the fraction

$$\frac{357294}{573440},$$

or, dividing both terms by 14,

$$\frac{25521}{40960}.$$

This reduced to a decimal gives

·6230712890625.

The operation, in the form indicated above, will be as on the margin. But as the adding of the successive numbers, in the several denominations, lengthens the operation, these may be added in at once, as the work proceeds, without assigning a separate line to each addition. The process will then have the neater form given on the margin. The quotient of 3·5 by 4 is first found, and then 10 is added; also before adding 23, the first figure, 6, of the quotient is obtained, and so on.

4	14·0 drs.
4	3·500
	0·875
	10
4	10·87500 oz.
4	2·7187500
	0·6796875
	23
4	23·679687500 lbs.
7	5·919921875
	0·845703125
	1
4	1·84570312500 qrs.
	0·46142578125
	12
20	12·461425781250 cwts.
	·6230712890625 tons.

4	14·0
4	3·5
4	10·87500
4	2·7187500
4	23·6796875
7	5·919921875
4	1·84570312500
20	12·461425781250
	·6230712890625

EXERCISES.

	Numbers to be reduced.	
1.	19s.	to the decimal of £1.
2.	9s. 6d.	£1.
3.	5s. $10\frac{1}{2}$d.	£1.
4.	15s. $9\frac{3}{4}$d.	£1.
5.	3d.	£10.
6.	10s. 6d.	12s. 6d.
7.	3 r. 11 poles	1 acre.
8.	3 cwt. 1 qr. 7 lbs.	1 ton.
9.	8s. $7\frac{1}{4}$d.	1 guinea.
10.	11 dwts.	1 lb. troy.
11.	10 drams	1 lb. avoird.
12.	4s. $7\frac{9}{11}$d.	£1.
13.	1 acre.	a square mile.
14.	7 fur. 25 poles.	1 mile.
15.	7 oz. 4 dwts.	1 lb. troy.
16.	17 h. 39 m. 45 s.	1 day.
17.	3 qrs. 3 lbs. 1 oz. $12\frac{4}{9}$ drs.	1 cwt.
18.	4 lbs. 8 oz. troy	10 lbs. avoird.
19.	3 fur. 8 p. 4 yds. 1 ft.	1 mile.
20.	7 bush. 2 pks. 1 gal.	1 qr.
21.	1 lb. troy	1 lb. avoird.
22.	£$\frac{7}{8}$—gn. $\frac{7}{8}$	a crown.
23.	5 h. 48 m. 49·6 s.	a day.
24.	30′ 45″	1 degree.
25.	1 oz. avoird.	1 oz. troy.

13. The process by which any given decimal is converted into an equivalent fraction embraces three cases, according as the decimal is *terminate*, a *pure circulator*, or a *mixed circulator*.

1. If the given decimal be terminate, it is of course only necessary to supply the denominator and reduce the result to its lowest terms.

Thus, $$\cdot 6875 = \frac{6875}{10000} = \frac{11}{16}.$$

2. In the case of a pure circulating decimal, make the period the numerator, and subscribe for denominator a number consisting of as many 9s as there are figures in the period.

Thus, $$\cdot\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}.$$

For, since $\cdot 111 = \frac{1}{9}$, $\cdot 0101 = \frac{1}{99}$, $\cdot 001001 = \frac{1}{999}$, $\cdot 00010001 =$

$\frac{1}{999999}$, &c., it is manifest that $\cdot\dot{6} = \frac{6}{9} = \frac{2}{3}$; $\cdot\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$; $\cdot\dot{5}6\dot{7} = \frac{567}{999} = \frac{189}{333} = \frac{21}{37}$; $\cdot\dot{3}8461\dot{5} = \frac{384615}{999999}$, which, by reduction by 76923, the greatest common measure of the terms becomes $\frac{5}{13}$.

Or the same may be shown otherwise, thus :—

Let the given decimal be called x, so that $x = \cdot666\ldots$

then (Note 2) $10x = 6\cdot666..$

hence $10x - x = 6\cdot666.. - \cdot666..$

or $9x = 6$

whence $x = \frac{6}{9} = \frac{2}{3}$.

Again, let the given decimal have two figures in the period, as $\cdot\dot{5}\dot{4}$, and let $x = \cdot5454..$; then, in order to obtain a whole number, we must multiply by $100 = 10^2$;

therefore $100x = 54\cdot5454..$

and $100x - x = 54\cdot5454.. - \cdot5454..$

or $99x = 54$

and therefore $x = \frac{54}{99} = \frac{6}{11}$.

Lastly, let the decimal have six figures in the period, as $\cdot\dot{3}8461\dot{5}$, and let $x = \cdot384615384615\ldots$; then it is obvious that we must use as multiplier the sixth power of 10;

hence $1000000x = 384615\cdot384615\ldots$

and $1000000x - x = 384615\cdot384615.. - \cdot384615..$

or $999999x = 384615$

and therefore $x = \frac{384615}{999999} = \frac{5}{13}$, as before.

3. The reduction of mixed circulating decimals will be effected by the same method. Let the decimal be multiplied by such powers of 10 as will render the non-circulating part, and also the entire mixed decimal to the end of the first period, whole numbers; then, by subtraction, we shall get quit of the decimal part, as already shown.

Thus, let the given decimal be $\cdot1\dot{6}$, and put $x = \cdot1666..$; then, since there is *one* figure in the non-circulating part, and *two* figures embrace the period, multiply by the *first* and *second* powers of 10; we thus have

$10x = 1\cdot666..$

and $100x = 16\cdot666..$;

hence $100x - 10x = 16\cdot666.. - 1\cdot666..$

or $90x = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$.

Again, let the decimal be $\cdot24\dot{5}\dot{7}$, and put $x = \cdot245757..$;

then the multipliers will be the second and fourth powers of 10; we thus have

$$100x = 24{\cdot}5757\,..$$

and $$10000x = 2457{\cdot}5757\,..$$

hence $$10000x - 100x = 2457{\cdot}5757\,..\ - 24{\cdot}5757\,..$$

or $$9900x = 2457 - 24$$

and $$x = \tfrac{2457-24}{9900} = \tfrac{2433}{9900} = \tfrac{811}{3300}.$$

Lastly, let the given decimal be $\cdot 012\dot{3}\dot{6}$, and put $x = \cdot 0123636\,..$;

then $$1000x = 12{\cdot}3636\,..$$

and $$100000x = 1236{\cdot}3636\,..$$

therefore $$99000 = 1236 - 12$$

and $$x = \tfrac{1236-12}{99000} = \tfrac{1224}{99000} = \tfrac{153}{12375}.$$

The same may also be shown in the following manner :—

Let the given decimal be $\cdot 4\dot{6}1\dot{8}$; then $\cdot 4\dot{6}1\dot{8} = \frac{4{\cdot}\dot{6}1\dot{8}}{10} = \frac{4 + \frac{618}{999}}{10} = \frac{4 \times 999 + 618}{9990}$.

Now, since $999 = 1000 - 1$, $4 \times 999 = 4 \times (1000 - 1) = 4 \times 1000 - 4$; therefore,

$$\frac{4 \times 999 + 618}{9990} = \frac{4 \times 1000 - 4 + 618}{9990} = \frac{4618 - 4}{9990};$$

which is the same as the result obtained by the other method. It is manifest that, if the non-circulating part consisted of two figures, the divisor would be 100, and the denominator would have two zeros, and so of others.

From a review of these operations we derive the following

Rule.—Make the difference between the decimal to the end of the first period, and the figures which do not circulate, taken as whole numbers, the numerator of the required fraction; and for denominator, take the number consisting of as many 9s as there are figures in the circulating part, followed by as many *zeros* as there are figures in the non-circulating part.

Thus, in the foregoing examples, the several numerators are $16 - 1$, $2457 - 24$, $1236 - 12$, the numbers being taken as integers; and the denominators 90, 9900, and 99000. Thus also, if the given decimal be $\cdot 62\dot{8}3\dot{8}$, the numerator will be $62838 - 62$, and the denominator 99900.

EXERCISES.

DECIMALS TO BE REDUCED.

1. $\cdot\dot{7}$.
2. $\cdot1\dot{4}$.
3. $\cdot13\dot{8}$.
4. $\cdot\dot{1}4\dot{8}$.
5. $\cdot013\dot{8}$.
6. $\cdot47\dot{5}4\dot{3}$.
7. $\cdot4\dot{9}$.
8. $\cdot044\dot{3}$.
9. $\cdot6\dot{5}3\dot{1}$.
10. $\cdot\dot{5}7142\dot{8}$.
11. $\cdot6\dot{4}2857\dot{1}$.
12. $\cdot5\dot{6}8\dot{1}$.

ADDITION AND SUBTRACTION.

14. From the nature of the notation, as already explained, it is obvious that the addition and subtraction of decimals will be performed exactly as in whole numbers.—The decimal points being placed in one vertical line, the digits of the same order are under one another. In Ex. 4 we conceive ciphers annexed after the 8, according to Art. 6.

In the case of interminate decimals, the required result must of course be limited to a certain number of places; and in order that the answer may be correct to the last place, it is necessary to carry out the repeating figures one or two places farther, increasing by a unit the last figure, if it be 5 or greater than 5; one or two figures are then rejected from the answer, which is therefore said *to be true* to the required number of places. In many cases the periodic figures into which the result divides itself will be readily discovered by carrying out the repeating figures a few places.

(1.)	(2.)
	14·2875
	12·676
23·875	8·4
15·648	41·0126
39·523, *sum.*	76·3761, *sum.*

(3.)	(4.)
16·5326	32·328
9·8457	18·75824
diff. 6·6869	*diff.* 13·56976

Ex. 5. Add together $15\cdot4\dot{2}\dot{7}$, $12\cdot41\dot{6}$, $11\cdot\dot{3}2\dot{8}$, $16\cdot\dot{0}2\dot{9}$, and $10\cdot\dot{4}7\dot{6}$, true to five places of decimals.

We here extend the periodic and repeating decimals a few places farther than the required number of places, increasing the last figures, when the next would be 5, or greater than 5.

15·42727273
12·41666667
11·32832833
16·02929293
10·47647648
65·67803714
65·67804, *Ans.*

Ex. 6. Add together 8·583′, 17·7′4′, 112·0′8′, 6·125, 15·6′, and ·76′9′.

The repeating figures are here carried out to eight places, and the required answer is 160·972979797, &c., that is 160·9729′7′, or 160·97298.

$$\begin{array}{r} 8{\cdot}58333333 \\ 17{\cdot}74747474 \\ 112{\cdot}08080808 \\ 6{\cdot}125 \\ 15{\cdot}66666667 \\ {\cdot}76969697 \\ \hline 160{\cdot}972979797, \text{ \&c.} \end{array}$$

Ex. 7. From $8{\cdot}\dot{5}4\dot{2}$ subtract $2{\cdot}\dot{9}\dot{5}$.

In this example the periods are extended to 9 or 12 places, and it is seen that the answer forms a period of 6 figures, being $5{\cdot}\dot{5}8294\dot{6}$.

EXERCISES.

1. 3·7812 + 4·00756 + ·010774 + 6 + 18·1 + 3·9375.
2. 156·1 + 312 + 441·001 + ·125 + ·00625 + 3·818475.
3. 14·84 + 7·1875 + 100 + ·22938625 + 4·1178342 + 1·0004.
4. 456 + 67·8 + 9·56 + ·478 + ·0687 + ·00932 + ·189765.
5. ·786425 + ·975324 + ·176009 + ·32 + ·62519375 + ·4.
6. Add together $3{\cdot}\dot{7}\dot{6}$, $4{\cdot}41\dot{6}$, $7{\cdot}\dot{4}\dot{5}$, $8{\cdot}\dot{3}8\dot{7}$, true to 5 places.
7. Add together $18{\cdot}1\dot{2}7\dot{6}$, $11{\cdot}\dot{3}4\dot{9}$, $12{\cdot}1\dot{4}\dot{5}$, $8{\cdot}\dot{6}4\dot{8}$, $15{\cdot}2\dot{3}$ true to 6 places.
8. Add together 15·7′, 12·4′, 18·3′87′, 5·416′, 10·746′87′, 14·9′, 23·4′5′, $4{\cdot}12\dot{3}4\dot{5}$, true to 6 places.
9. Add together ·6′, ·83′, ·8′57142′, ·75, and ·8.
10. Add together ${\cdot}4\dot{6}32\dot{1}$, ${\cdot}81\dot{5}3\dot{2}$, ${\cdot}1549\dot{2}\dot{6}$, ${\cdot}75\dot{3}\dot{2}$.
11. From 12·73 subtract $5{\cdot}\dot{1}\dot{7}$.
12. „ 1 subtract ·0625.
13. „ 16·008576 take 11·789.
14. „ ·01 take ·001.
15. „ 513·5 subtract 258·764085.
16. „ 10·8 take $4{\cdot}37546\dot{5}\dot{8}$.
17. „ 18·375 subtract $6{\cdot}\dot{4}2857\dot{1}$.
18. „ ·7456 take ${\cdot}357\dot{3}2978\dot{4}$.
19. „ $12{\cdot}46\dot{5}7\dot{4}$ take $7{\cdot}18\dot{3}4\dot{5}$.
20. „ ${\cdot}835\dot{7}2\dot{4}$ subtract ${\cdot}37\dot{6}\dot{8}$.
21. „ ${\cdot}\dot{8}4\dot{5}$ subtract ${\cdot}\dot{3}\dot{7}$.
22. „ ${\cdot}576\dot{5}43\dot{8}$ take ${\cdot}3\dot{2}168\dot{7}$.

MULTIPLICATION.

15. The process of multiplying a decimal by a whole number is exactly the same as in the case of integers. Thus, in the annexed example, we carry as in whole numbers, and there is no difficulty in placing the point. But when there are decimals in both factors regard must be had to the denominators. If we multiply *tenths* by *tenths*, the answer must be *hundredths; if hundredths* by *tenths*, the answer must be *thousandths*, and so on. Thus, ·2 × ·3 = ·06, ·132 × ·24 = 3168 *hundred thousandths* $= \frac{3168}{100000}$, in writing which without denominator, the last figure must occupy the fifth place, so that a cipher is prefixed, and the product written ·03168. In general, since the product of the denominators must contain as many ciphers (Note 3, *b*) as there are in all the separate denominators, and since the number of places after the point in any decimal is the same (Art. 3) as the number of ciphers in the denominator, it follows that the product of any number of decimals will contain as many decimal places as there are in all the factors. In the first of the above examples there are two decimal places in the product, one for each of the factors; in the second, the one factor has *three*, the other *two*, and the product *five* places. And if we multiply together continually ·3, ·12, and ·014, we must have *six* decimal places in the answer; for the denominator of the product will contain *six* ciphers, because the denominator of the first factor has *one* cipher, that of the second *two*, and that of the third *three*. The product, as denoting 504 *millionths*, will therefore be written ·000504. We have, therefore, the following

$$\begin{array}{r} 7\cdot658 \\ 4 \\ \hline 30\cdot632 \end{array}$$

Rule.—To multiply decimals, proceed as in simple multiplication, and point off in the product as many decimal places as there are in all the factors; supplying any deficiency by prefixing ciphers.

Ex. 1. Multiply ·035274 by 7000.

We here remove the decimal point three places to the right, and then multiply by 7.

$$\begin{array}{r} 35\cdot274 \\ 7 \\ \hline 246\cdot918 \end{array}$$

Ex. 2. Multiply ·674 by 34·6.

Here we proceed as in multiplying integers, and point off 4 places in the answer.

$$\begin{array}{r} \cdot674 \\ 34\cdot6 \\ \hline 4044 \\ 2696 \\ 2022 \\ \hline 23\cdot3204 \end{array}$$

Ex. 3. Multiply 4·$\dot{5}\dot{6}$ by 2·43.

Here, multiplying by 3 we should have 18, but as the next figure of the period is a 5, there is 1 to carry, and we have therefore 19; again, in multiplying by the second figure 4, we must, in like manner, carry 2, making 26, 6 being put down in the second place, and 2 carried; and it is obvious that a 2 must be put under the 9 and not a 0, as a 2 is carried from a succeeding 6. The process will be better understood from the work on the margin carried out four places farther.

$$\begin{array}{r} 4\cdot\dot{5}\dot{6} \\ 2\cdot43 \\ \hline 1369 \\ 18262 \\ 91313 \\ \hline 11\cdot094\dot{5} \end{array}$$

$$\begin{array}{r|l} 4\cdot56 & 5656 \\ 2\cdot43 & \\ \hline 1369 & 6969\,.\,. \\ 18262 & 6262\,.\,. \\ 91313 & 1313\,.\,. \\ \hline 11\cdot0945 & 4545\,.\,.\,. \end{array}$$

Ex. 4. Multiply ·6$\dot{7}$89$\dot{5}$ by 4·632.

In this example, 1 is carried to the product of 2 and 5, the next figure being 7; likewise to the product of 3 and 5, 2 is carried. The figures which occupy the places where ciphers stand in simple multiplication, and which are here printed in a smaller type, are supplied from our knowledge of the succeeding figures of the period. To show the process by which these figures are obtained, the work is again given on the margin, with an additional period of the circulator. It is, however, better to work such examples by the CONTRACTED METHOD, explained in the next article.

$$\begin{array}{r} \cdot6\dot{7}89\dot{5} \\ 4\cdot632 \\ \hline 135791 \\ 203687{\scriptstyle 8} \\ 407374{\scriptstyle 78} \\ 271583{\scriptstyle 158} \\ \hline 3\cdot1449\dot{3}29\dot{7} \end{array}$$

$$\begin{array}{r|l} \cdot67895 & 7895\,.\,. \\ 4632 & \\ \hline 135791 & 5791\,.\,. \\ 2036873 & 687{\scriptstyle 8}\,.\,. \\ 40737473 & 74{\scriptstyle 78}\,.\,. \\ 271583158 & 3{\scriptstyle 158}\,.\,. \\ \hline 3\cdot14493297 & 3297\,.\,. \end{array}$$

EXERCISES.

1. Multiply 6·27 by 15·9.
2. „ 11·4 by 1·14.
3. „ 1240 by ·008.

4. Multiply ·156 by ·143
5. ,, 375·4 by ·057.
6. ,, 1500 by ·008.
7. ,, ·06 by 3000.
8. ,, ·002 by 400.
9. ,, 164·3715 by 29·46375.
10. ,, ·111 by 11·1.
11. ,, 175·125 by 15·625.
12. ,, ·0021 by 21.
13. ,, 464·3 by ·00521.
14. ,, 12·5 by ·062216.
15. ,, ·398765 by 10100
16. ,, 34·124 by ·0051.
17. ,, 7·$\dot{4}\dot{3}$ by ·862.
18. ,, 73·6$\dot{3}$7$\dot{1}$ by 8·143.
19. ,, ·4$\dot{8}$43$\dot{7}$ by ·5734.
20. ,, 32·$\dot{3}$7$\dot{4}$ by 1·264.
21. ,, 6·$\dot{5}\dot{8}$ by 3·74.
22. Multiply together continually 1·2, 3·25, and 2·125.
23. Find the continued product of 6400, ·008, ·04, and ·2.
24. Find the continued product of 11, 1·1, ·11, ·011, and ·101.
25. Multiply together continually 500000, ·01, and ·0003.

CONTRACTED METHOD.

16. It often occurs in arithmetical computations that we are required to multiply together numbers consisting of many figures, and yet the answer can be found with sufficient exactness by means of a few figures of the product. Thus, into the calculation of compound interest and annuities, such numbers constantly enter as high powers (13th, 14th .... 21st) of 1·05, 1·04, 1·045, the labour of finding which, in their complete form, would be immense; while four or five figures are all that are required to insure accuracy; since, if our unit is £1, an error of 1 in the third decimal place ($\frac{1}{1000}$) would be under a farthing ($\frac{1}{960}$). Also, in multiplying together recurring decimals, we use numbers which are only approximately correct; we are obliged to employ them in an incomplete form because of the interminable series in which they extend.

Now, interminate decimals being far more numerous than those which are terminate, such cases will be constantly occurring, and it is most desirable to have a method whereby we can ascertain a few figures as accurately as possible, and then dismiss from our computation those in which no confidence can be placed. The method of contracted multiplication has been devised by arithmeticians for this purpose.—It proceeds upon the following principles:

17. In simple multiplication the figures of the multiplier may be taken in any order, provided the partial products be so set down for addition, that digits of the same order may be in one vertical line. The following arrangements illustrate this:—

$$\begin{array}{r} 5372 \\ 1634 \\ \hline 21488 \\ 16116 \\ 32232 \\ 5372 \\ \hline 8777848 \end{array} \qquad \begin{array}{r} 5372 \\ 1634 \\ \hline 16116 \\ 32232 \\ 21488 \\ 5372 \\ \hline 8777848 \end{array} \qquad \begin{array}{r} 5372 \\ 1634 \\ \hline 5372 \\ 32232 \\ 16116 \\ 21488 \\ \hline 8777848 \end{array}$$

The third form is that which the process will take when the multiplier is set down in an inverted order, as on the margin. This arrangement is more convenient, inasmuch as the multiplication then proceeds in the usual way, from right to left. It is adopted in the CONTRACTED METHOD, for reasons which will immediately appear.

$$\begin{array}{l} 5372 \\ 4361 \\ \hline 5372 \\ 32232 \\ 16116 \\ 21488 \\ \hline 8777848 \end{array}$$

The work by this contracted method proceeds according to the following

Rule.—(1.) Count off in the multiplicand, to the right of the decimal point (annexing ciphers if necessary), as many decimal places as are required in the product; (2.) under the last of these put the unit figure of the multiplier, or a cipher if there be no unit figure; and write its other figures in reversed order; (3.) multiply by each figure of this reversed multiplier, neglecting all the figures in the multiplicand which lie to the right of each figure multiplied by, except for the purpose of ascertaining how much is to be *carried* from these figures; (4.) arrange the par-

tial products so that their first figures shall be in the same vertical column, and from their sum cut off the number of decimal places required.

Multiply 6·35642 by 47·6453, limiting the product to three decimal places.

Here the multiplier is written in an inverted order, and the unit figure 7 is placed under the *third* decimal figure of the multiplicand, as the product is limited to *three* places. We then multiply by 4, neglecting the 2, but carrying 1 to the 16, since 8 (4 × 2) is nearer 10 than 0. Next, we multiply 6 by the 7 standing under it, carrying 3, as the preceding product would be 29; and so with the others. As the last figure 3 has no figure above it, the product is 0, but we carry 2, the product in the place preceding being 19.

$$\begin{array}{r}
6\cdot35642 \\
354674 \\
\hline
254257 \\
44495 \\
3814 \\
254 \\
32 \\
2 \\
\hline
302\cdot854
\end{array}$$

It is merely a matter of convenience thus to reverse the figures of the multiplier, as it saves trouble in determining what figure of the multiplicand is to be used with the successive figures of the multiplier. The process has the same form when these figures are written in direct order, as on the margin, but we proceed from left to right in taking the figures of the multiplier. We begin with the fourth decimal figure 4, multiplying it by the first figure 4 of the multiplier, and carry 1 for the preceding product, 8; we next multiply by the second figure 7, beginning with the third figure 6 of the multiplicand, carrying 3, and so on.

$$\begin{array}{r}
635642 \\
476453 \\
\hline
254257 \\
44495 \\
3814 \\
254 \\
32 \\
2 \\
\hline
302\cdot854
\end{array}$$

The reason of the operation will be understood from the work of the preceding example, given at full length both in the common form, and with the figures of the multiplier reversed.

$$\begin{array}{r|l}
\multicolumn{2}{r}{6\cdot35642} \\
\multicolumn{2}{r}{47\cdot6453} \\
\hline
1 & 906926 \\
31 & 78210 \\
254 & 2568 \\
3813 & 852 \\
44494 & 94 \\
254256 & 8 \\
\hline
302\cdot853 & 537826
\end{array}
\qquad
\begin{array}{r|l}
\multicolumn{2}{l}{6\cdot35642} \\
\multicolumn{2}{l}{3546\cdot74} \\
\hline
254256 & 8 \\
44494 & 94 \\
3813 & 852 \\
254 & 2568 \\
31 & 78210 \\
1 & 906926 \\
\hline
302\cdot853 & 537826
\end{array}$$

The answer by the contracted method comes out exactly what we should have made it, by the addition of 1 to the last figure on account of the 5 following.—When we know from the nature of the computation in which we are engaged how many figures are required, it is better to work for an additional figure or two, and then to reject them after addition. This is especially necessary in such calculations as are referred to in the beginning of the last article.

The full operation also shows that the first figures of the contracted work, standing in one vertical column, are all of the same order in the decimal scale; in this case *thousandths.* That they must be of the same order will also appear when we consider, that each successive figure of the multiplicand is of a higher order, and each in the multiplier of a lower order, and therefore all the first figures are of the same order as that figure of the multiplicand which stands over the units of the multiplier.

EXERCISES.

1. Multiply 31·72164 by 1·72196, so as to have three decimal places in the product.

2. Multiply 5·79432 by ·643242, retaining five decimal places.

3. Multiply $7\cdot\dot{2}8571\dot{4}$ by 36·74405, to six decimal places.
4. Multiply 3·7719214 by ·4471618, limiting to six places.
5. Multiply ·53217 by ·63942, to five places.
6. Multiply 3·8640372 by 1·2479603, to seven places.
7. Find the second power of ·35622, to five places.
8. Multiply $1\cdot\dot{6}\dot{5}$ by $1\cdot\dot{4}\dot{8}$, to five places.

DIVISION.

18. In dividing a decimal by a whole number, the successive remainders are carried to the digits next following, as in simple division, and the position of the separating point is easily fixed. Thus, the quotient of 43·725 by 6 is 7·2875, a cipher being annexed to the remainder 3 (Art. 6), and that of ·104824 by 12 is ·0087353′. But if the divisor be a decimal, apply the following

Rule.—To divide by a decimal:—(1.) If the number of decimal places be not the same in the divisor and dividend, annex ciphers or periodic figures to supply the deficiency; (2.) reject the points; (3.) divide as in whole numbers, and the quotient will be a whole number; (4.) if there be a remainder, annex ciphers or periodic figures, and the decimal part of the quotient will be obtained.

The reason of this rule will be readily understood from the following example:—

Ex. 1. Let it be required to divide 220·9375 by 17·5.

When we annex three ciphers to the divisor, which does not alter its value (Art. 6), it becomes 17·5000, and the fractional parts have now the same denominator. Supplying this denominator, and reducing the mixed numbers to a fractional form, we have $\frac{175000}{10000}$ and $\frac{2209375}{10000}$. Now, when two fractions have the same denominator, the one fraction is contained in the other as often as the numerator of the former is contained in the numerator of the latter; we therefore reject the denominators, and divide as in whole numbers. The rest of the process is the same as in Reduction (Art. 7). There are thus two steps in the process: (1.) making the number of decimal places the same, that is, making the denominators the same; and (2.) *rejecting* these equal denominators.[4]

$$\begin{array}{r}
17{\cdot}5)\,220{\cdot}9375 \\
17{\cdot}5000)\,220{\cdot}9375 \\
17\tfrac{5000}{10000})\,220\tfrac{9375}{10000} \\
\tfrac{175000}{10000})\,\tfrac{2209375}{10000}
\end{array}$$

$$\begin{array}{rl}
175000) & 2209375\,(12{\cdot}625 \\
 & \underline{175000} \\
 & 459375 \\
 & \underline{350000} \\
 & 1093750 \\
 & \underline{1050000} \\
 & 437500 \\
 & \underline{350000} \\
 & 875000 \\
 & \underline{875000} \\
 & \cdots\cdots
\end{array}$$

---

[4] By attending to the following considerations, the division may be performed with the quantities as given, without first making the number of decimal places the same. For, since, as has been shown under Multiplication (Art. 15), the dividend, or given product, must contain as many decimal places as there are in the divisor and quotient, that is, in both factors, it is obvious that the number of decimal places in the quotient will be equal to the excess of the number in the dividend above the number in the divisor, all the ciphers or periodic figures annexed in the course of the operation, *counting as decimal places of the dividend.* If there be the same number in the divisor and dividend, the quotient will be a whole number, or will contain no decimal places; but if the number in the divisor exceed the number in the dividend, this excess *must be reckoned as less than nothing*, or *a negative quantity*; that is, instead of *cutting off* decimal places in the quotient, ciphers *must be annexed to it* to make up the deficiency. Thus, in the first of the examples in the text the divisor contains *one* decimal place, the dividend *four*, and the

Ex. 2. Divide 181·3 by ·0037.

We here annex three ciphers to the dividend and reject the points; the divisor then becomes 37; and the quotient is found, as in simple division, to be 49000.

$$\begin{array}{r}
\cdot0037)\,181\cdot3\\
\cdot0037)\,181\cdot3000\\
37)\,1813000\,(49000\\
148\\
\hline
333\\
333\\
\hline
000
\end{array}$$

$$\begin{array}{r}
8634)\,12757\,(1\cdot477597\ldots\\
8634\\
\hline
41235\\
34536\\
\hline
66997\\
60438\\
\hline
65595\\
60438\\
\hline
51577\\
43170\\
\hline
84075\\
77706\\
\hline
63697\\
60438\\
\hline
3259,\ \&c.
\end{array}$$

Ex. 3. Divide $12\cdot\dot{7}\dot{5}$ by 8·634.

In this example, the places are equalized by affixing the next figure of the period; and in carrying on the division, the periodic figures are in turn annexed to the successive remainders. The answer, found to six decimal places, is 1·477597 or 1·4776.

Ex. 4. Divide ·25 by $\cdot\dot{3}$.

The common operation gives here ·75, with a very small excess, which is rejected, because we know that the quotient must be ·75, for $\cdot\dot{3} = \frac{1}{3}$ and $\cdot25 = \frac{1}{4}$; and $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4} =$ ·75.

$$\begin{array}{rl}
3333)\,25000 & \left(0\cdot750009\right.\\
23333 & \left(0\cdot75\ \textit{Ans.}\right.\\
\hline
16670 & \\
16667 & \\
\hline
3 &
\end{array}$$

Ex. 5. Divide $5\cdot\dot{0}\dot{9}$ by $6\cdot\dot{2}$.

Here, extending the repeating figures to 4, 5, or 6 places, we find for quotient ·8181 . . or $\cdot\dot{8}\dot{1}$ a circulator.

quotient *three.* In Ex. 2, there are *three* decimal places more in the divisor than in the dividend, and after all the figures in the latter have been brought down, giving 49 for the significant figures of the quotient, *three* ciphers have to be annexed, making the quotient 49000.—Or the same may be shown thus: $181\cdot3 \div \cdot0037 = \frac{1813}{10} \div \frac{37}{10000} = \frac{1813}{10} \times \frac{10000}{37} = \frac{10000}{10} \times \frac{1813}{37} =$ $1000 \times 49 = 49000$. The position of the decimal point, however, may often be fixed by mere inspection. Thus, in Ex. 3, we see at once that the point must be placed after the 1 in the quotient.

Ex. 6. Divide $\cdot 08\dot{3}$ by $\cdot 1\dot{6}$.

Here, extending the repeating figures in each, we find ·5 for the quotient, without any remainder, as we carry 3 in multiplying the first 6 by 5. The given decimals being obviously the same as $\frac{1}{12}$ and $\frac{1}{6}$, we have $\frac{1}{12} \div \frac{1}{6} = \frac{6}{12} = \frac{1}{2} = \cdot 5$. It is usually preferable, however, when we have an interminate divisor, to reduce both decimals to fractions, and then to reduce the resulting fraction to a decimal. But when division is employed according to the preceding Rule, and the divisor contains many figures, the Contracted Method should be adopted, and the quotient limited to a few places.

EXERCISES.

1. Divide 70·8946 by 13·825.
2. „ 126029·501025 by 3706·205.
3. „ ·001304 by ·004.
4. „ ·22 by ·2.
5. „ 1·1 by ·11.
6. „ ·111 by ·3.
7. „ ·003125 by ·025.
8. „ 3·1 by ·0025.
9. „ 6 by ·008.
10. „ 1¼ by ·0125.
11. „ ·54439 by 7777.
12. „ ·322 by ·0007.
13. „ ·321 by 32·1.
14. „ 31½ by ·0126.
15. „ 52·65 by ·045.
16. „ 11·121 by 3·4571 true to five places.
17. „ 7984·32 by 9·324 true to four places.
18. „ 1·0072 by $\cdot\dot{0}\dot{7}$ true to five places.
19. „ $\cdot 6\dot{3}78\dot{2}$ by ·4623 true to six places.
20. „ $2\cdot\dot{3}7\dot{0}$ by $4\cdot\dot{9}2307\dot{6}$.
21. 123·70536 ÷ 54·25; 1 ÷ 3·1416; ·00735 ÷ ·01.
22. ·000004 ÷ ·004; ·326 ÷ ·0326; 12⅗ ÷ ·0012; 1 ÷ ·001.
23. 269·0625 ÷ 14·35; ·520773512 ÷ ·63478; 1857 ÷ ·00001.
24. Divide successively 17355, 1735·5, 173·55, 17·355 by 6·5.
25. Divide 17·328 by ·475, 4·75, 47·5, and 475 successively.

CONTRACTED METHOD OF DIVISION.

19. When the divisor contains many figures, and the quotient is required to a few places which can be depended upon, the following contracted method of division is employed:—

Rule.—(1.) Prepare the numbers, fix the position of the point, find the first figure of the quotient and first remainder, as in the common method (Art. 18); then, instead of annexing to this remainder the next figure of the dividend, *cut off the final figure of the divisor*, and use the preceding part to find the second figure of the quotient; (3.) in multiplying by this second figure, *take into account the figure cut off*, in order to know how much is to be carried; (4.) proceed in the same manner with the other figures of the divisor, cutting off one at each step till the divisor is exhausted, or the required number of figures is obtained.

Ex. 1. Divide 2·3896 by 1·4637.

$$\begin{array}{r r l}
14637) & 23896 & (1{\cdot}632 \\
\cdots & 14637 & \\ \cline{2-2}
 & 9259 & \\
 & 8782 & \\ \cline{2-2}
 & 477 & \\
 & 439 & \\ \cline{2-2}
 & 38 & \\
 & 29 & \\ \cline{2-2}
 & 9 &
\end{array}
\qquad\qquad
\begin{array}{r r|l}
14637) & 23896 & (1{\cdot}632 \\
 & 14637 & \\ \cline{2-3}
 & 9259 & 0 \\
 & 8782 & 2 \\ \cline{2-3}
 & 476 & 80 \\
 & 439 & 11 \\ \cline{2-3}
 & 37 & 690 \\
 & 29 & 274 \\ \cline{2-3}
 & 8 & 416
\end{array}$$

Here, instead of annexing a cipher to the first remainder 9259, the last figure of the divisor is cut off; then the second figure of the quotient being 6, in finding the second partial product 8782, we carry 4 (7 × 6 = 42) to 18 (3 × 6). Again, cutting off 3, the second figure of the divisor, and multiplying its third figure 6 by 3, the next quotient figure, we carry 1 from the preceding figure, and thus find the third partial product 439, and so on. To show the progress of the work, a dot or other mark should be placed below the figures of the divisor, as they are successively cut off. The uncontracted operation is given on the margin in order to show the *rationale* of the process more clearly.

Ex. 2. Divide ·4368215 by ·54637482 to 7 decimal places.

```
54637482 ) 436821500 ( ·7994905
  ······   382462374
           ---------
            54359126
            49173734
            --------
             5185392
             4917373
             -------
              268019
              218550
              ------
               49469
               49173
               -----
                 296
                 273
                 ---
                  23
```

Ex. 3. Divide $2\cdot\dot{3}7\dot{0}$ by $4\cdot\dot{9}2307\dot{6}$.

```
4923077) 23703703 ( ·481481.  (See Ex. 20, Art. 18.)
  ·····  19692308
         --------
          4011395
          3938462
          -------
            72933
            49230
            -----
            23703
            19692
            -----
             4011
             3938
             ----
               73
               49
               --
               24
```

EXERCISES.

1. 8·6134 ÷ 7·3524 to four decimal places.
2. 897·5462 ÷ 189·7654 to five decimal places.
3. $\cdot\dot{6}\dot{1} \div 13\cdot543516$.
4. $\cdot\dot{0}\dot{9} \div \cdot\dot{2}3076\dot{9}$
5. 7486·31472 ÷ 9·86487 retaining four decimal places.
6. $812\cdot54567 \div 7\cdot\dot{3}\dot{4}$.

20. *To find the value of a decimal in terms of the lower denominations of any given concrete unit:*—Multiply the given decimal (as in the reduction of compound quantities) by the number expressing how often each successive lower denomination is contained in the next higher, pointing off (Art. 15) as many figures in each product as there are places in the given decimal; the remaining figures towards the left will express the value required.

Ex. 1. Find the value of ·778625 of £1, or £0·778625.

Here, multiplying, as in Reduction, by 20, 12, and 4, the numbers expressing how often the successive lower denominations are contained in the next higher, we find for the required value 15s. 6$\frac{3}{4}$·48, or 15s. 6$\frac{3}{4}$· ·$\frac{12}{25}$.

```
 ·778625
      20
15·5725s.
     12
 6·87d.
    4
 3·48q.
```

Ex. 2. Find the value of ·65′7′ of 1 ton.

Here, multiplying by 10 and by 2, and carrying 1 as the next figure is 5, we find 13 cwt., ·1′5′; multiplying this by 4 we have the repeater ·6′0′. In multiplying this by 28 we carry 1 in the second line, as the succeeding figure is 6, and find the repeater ·9′6′. Again, in multiplying by 16 we carry 1 in the second line, as the next figure would be 6; and we thus obtain the repeating decimal ·5′1′. Lastly, multiplying by 16 we find 8 drs. with the remainder ·2′4′ = $\frac{8}{33}$d. The required value is therefore 13 cwt. 0 qr. 16 lbs. 15 oz. 8$\frac{8}{33}$ drs. Otherwise, ·65$\dot{7}$ = $\frac{651}{990}$ = $\frac{217}{330}$; then $\frac{217}{330}$ of 1 ton = $\frac{217}{330}$ tons = 13 cwt. 0 qr. 16 lbs. 15 oz. 8$\frac{8}{33}$ drs.; and this is perhaps the best mode in most cases.

```
 ·65′7′
   10
6·5′7′
    2
13·1′5′ or,

13·151515 cwt.
        4
 0·60606 qr.
      28
 4 84848
12 12121
16·96969 lbs.
      16
 5 81818
 9 6969
15·5151 oz.

15·51515
      16
 3 0909
 5 1515
 8·2424 drs.
```

Ex. 3. Find the value of £$\cdot 6\dot{1}\dot{8}$.

The successive decimals here found are periodic, and since $\cdot\dot{4}\dot{5}=\frac{45}{99}=\frac{5}{11}$, the answer is 12s. $4\frac{1}{4}\cdot\ \cdot\frac{5}{11}$. Otherwise, reduce $\cdot 6\dot{1}\dot{8}$ to a fraction by the Rule of Art. 13; this gives $\frac{612}{990}=\frac{34}{55}$; then $\frac{34}{55}$ of £1, or £$\frac{34}{55}$, is 12s. $4\frac{1}{4}\cdot\frac{5}{11}$., as before. It is often a convenient and exact mode to increase the last figure by 1, when it is 5 or greater than 5, and take the decimal as terminate: or to take the nearest value of the period; thus, for ·47916 write ·47917, and for ·61818 take ·6182.

$$\begin{array}{r}
\cdot 6\dot{1}\dot{8} \\
10 \\
\hline
6\cdot\dot{1}\dot{8} \\
2 \\
\hline
12\cdot\dot{3}\dot{6}\text{s.} \\
12 \\
\hline
4\cdot\dot{3}\dot{6}\text{d.} \\
4 \\
\hline
1\cdot\dot{4}\dot{5}\text{q.}
\end{array}$$

Ex. 4. Find the value of ·08 of £5 18s. 9d.

Here, £5 18s. 9d. = £5·9375; then, £5·9375 ÷ 100 = £·059375; and this multiplied by 8 gives £·475, which being reduced as in *Ex.* 1, gives 9s. 6d. for the value required. Or we may reduce the given quantity to the lowest denomination, multiply the result by the given decimal, and again reduce this product to the highest denomination.

Ex. 5. Find the value of $\cdot\dot{8}\dot{4}$ of 6s. 5d.

Here, $\cdot\dot{8}\dot{4}=\frac{84}{99}$; and $\frac{84}{99}$ of 6s. 5d. $=\frac{84}{99}$ of 77d. $=\frac{84}{9}$ of 7d. $=\frac{7}{9}$ of $84=9\frac{1}{3}$d. $\times 7=65\frac{1}{3}$d. $=$ 5s. $5\frac{1}{4}\cdot\frac{1}{3}$.

EXERCISES.

Quantities to be valued.

1. £·76875.
2. £·3865.
3. £$\cdot 97291\dot{6}$.
4. ·05 of an acre.
5. ·0625 of an acre.
6. ·2674 of an acre.
7. ·375 of a guinea.
8. ·2375 of a guinea.
9. ·6845 of a cwt.
10. ·165625 of a ton.
11. ·315 of 2 lbs. 7 oz. 15 dwts.
12. ·047460 9375 of £10 13s. 4d.
13. $\cdot\dot{4}$ of 9s. $4\frac{1}{2}$d.
14. $\cdot 2\dot{1}\dot{7}$ of 8s. 3d.
15. $\cdot 3\dot{4}0\dot{5}$ of 2 cwt. 0 qr. 22 lbs. 10 oz. $10\frac{2}{3}$ drs.
16. ·002739 of a day.
17. ·98565 of 1°.
18. $\cdot 0\dot{2}\dot{6}$ of 1° 15′.
19. $\cdot\dot{1}\dot{2}$ of 13s. 9d.
20. $\cdot\dot{1}4\dot{8}$ of 3 qrs. 24 lbs.
21. $\cdot\dot{1}2\dot{3}$ of 115 lbs. 8 oz. 17 dwts. 18 grs.
22. $\cdot\dot{2}8571\dot{4}$ of a cwt.
23. $\cdot\dot{1}4285\dot{7}$ of 3 guineas.
24. $\cdot 16\dot{3}$ of $2\frac{1}{2}$ miles.
25. ·03756 of a century.

DECIMAL COINAGE.

APPLICATION OF DECIMALS.

21. Many of the processes of commercial arithmetic are greatly abridged by using decimals; and if the decimal subdivision of the pound sterling, partly effected some years ago by the issue of the florin, were completed, the advantage gained by the use of decimals would be so obvious as to lead to their more general adoption in all mercantile calculations. With our present complicated system of weights and measures, this change would not give all the advantages which would otherwise flow from it; but if a good metrical system were devised upon a decimal plan, the benefit would be very great; as those processes of arithmetic now the most lengthened and troublesome would be reduced to operations under the four "Simple Rules." The adoption of some such metrical system will probably soon follow on the introduction of a decimal coinage; but even without it the latter would afford an immense facility in reckoning, by simplifying and abridging many arithmetical processes.—We purpose, in what follows, to illustrate some of these abbreviated methods, and the application of a decimal coinage to mercantile calculations. To shorten these as much as possible, several tables are subjoined.

22. The plan of a decimal coinage, recommended by a Parliamentary Committee, and received with general favour throughout the country, proposes to retain the pound sterling, or sovereign, as the unit, so that the florin or two-shilling piece, now in circulation, will be one-tenth of the unit. Thus, £1 2s. will be decimally 1·1; £1 3s. will be 1·15, since a 5 in the second place is the half of a 1 in the first place; £3 16s. will be 3·8; £3 17s. will be 3·85, and so on. Only two other coins are required to complete the system; one, equal in value to the tenth of a florin or the hundredth of £1, which it is proposed to call a *cent;* and another, one-tenth of the cent, one-hundredth of the florin, or one-thousandth of the pound, for which the name *mil* has been proposed. The

three denominations inferior to the unit are thus tenths, hundredths, thousandths, so that £72, 6 florins, 8 cents, 5 mils, will be written decimally £72·685. The value of the cent, in terms of our present coinage, will therefore be $\frac{24}{10}$d. = $2\frac{4}{10}$d.; and since the pound contains 960 farthings, 1 mil = $\frac{960}{1000}$ = $\frac{96}{100}$ = $\frac{24}{25}$ of a farthing, or $\frac{24}{100}$d.; that is, 25 mils make 24 farthings or 6d., or the mil is 4 per cent. less than the farthing. Also, 1 farthing = $1\frac{1}{24}$ mils. These will be the *coins of account*, that is, the legal denominations of money, in which accounts will be kept, as we now keep them in pounds, shillings, pence, and farthings. But it is proposed that there shall be other coins in circulation, just as we now have half-sovereigns, crowns, half-crowns, &c. Of our old coins it would be convenient to retain those which are aliquot parts of the new coins; such as the half-sovereign, crown, shilling, and sixpence.

We have then—

10 mils = 1 cent (*ct.* or *c.*)
10 cents = 1 florin (*fl.* or *f.*)
10 florins = £1 (one pound sterling).
Also, 1 mil = $\frac{24}{25}$ farth. = $\frac{24}{100}$d.; 1 farth. = $1\frac{1}{24}$ mils.
1 pound = 10 florins = 100 cents = 1000 mils (*ml.* or *m.*)
1 shilling = 50 mils; 2 shillings = 1 florin = 100 mils.
1 sixpence = 25 mils.

It is manifest that £24 7fl. 4ct. 3m., or £24·743, may also be read £24 and 743 mils, 247 florins 43 mils, 2474 cents 3 mils, or lastly, 24,743 mils. Thus also, 56,527 mils = £56 5fl. 2ct. 7m.; 18,304 mils = £18 3fl. 4m., and £5 4fl. 6m. = 5406 mils. There is thus no necessity for successive reductions; the numbers are merely annexed to one another; or, if given in the lower denomination, are written out separately, all the figures beyond the third place expressing pounds. Neither is there any necessity to introduce the decimal point; the money may be at once set down as mils, and operated with independently of any consideration regarding decimals. But the most convenient and scientific mode will be to place the point after the pounds, and call the figures to the right mils, or florins, cents, and mils,—while we think of them as decimals of the pound, in performing additions, subtractions, &c.

EXAMPLES.

Ex. 1. Add together £43, eight florins, and seventeen mils; £15, seven florins, and nine mils; three hundred and forty-five florins, six cents, and eight mils; thirty-three thousand five hundred and ninety-four mils; five thousand six hundred and forty-six cents, five mils; £24, seven florins, eight cents, and six mils.

These quantities are either arranged as in addition of decimals, or are set down as mils, the former mode being the best. They might also be set down as florins and mils, or as cents and mils, as above explained.

(1.)	(2.)
43·817	43817
15·709	15709
34·568	34568
33·594	33594
56·465	56465
24·786	24786
£208·939	208939m.

That is, £208 9fl. 3ct. 9 m.

Ex. 2. Add together 3fl. 8ml.; 9fl. 3c.; 320c., $18\frac{3}{4}$ml.; 3c. $7\frac{5}{8}$ml.; 3275ml.; £2 and $11\frac{1}{2}$ml.; 1fl. $32\frac{3}{4}$ml.; 2c. $6\frac{1}{4}$ml.

In this example, the fractions of the mils being carried out decimally, the answer is found to be £9 9fl. 3ct. $9\frac{7}{8}$ml.

£0·308
0·930
3·21875
0·037625
3·275
2·0115
0·13275
0·02625
£9·939875

Ex. 3. From £7 5fl. 8c. 6ml. take 5798ml., and from £3 42·6ml. take 20·68fl.

Here, the numbers being arranged as pounds and mils, the process is seen to be exactly the same as in subtraction of decimals.

7·586	3·0426
5·798	2·068
£1·788	£0·9746

Ex. 4. Multiply £546 19s. 9d. by 328.

Here the decimal equivalent of the shillings and pence is £·9875; then £546·9875 × 328 = £179411·9 = £179411 18s. 0d.

Ex. 5. Multiply £458 7fl. 8ct. 6ml. by $57\frac{3}{4}$.

Here £458·786 × 57·75 = £26494·8915 = £26494 8fl. 9ct. $1\frac{1}{2}$ml.

Ex. 6. Multiply £7367 18s. $10\frac{3}{4}$d. by $197\frac{5}{8}$.

Taking the decimals, and using the Contracted Method, so as to obtain four decimal places, the answer is found to be £1456090 1s. $9\frac{1}{4}\cdot\frac{11}{25}$.

$$\begin{array}{r}
7367{\cdot}9447916 \\
526791 \\
\hline
7367944792 \\
6631150312 \\
515756135 \\
44207669 \\
1473589 \\
368397 \\
\hline
1456090{\cdot}089\,|\,4
\end{array}$$

Ex. 7. Divide £7545 8f. 2c. 5m. by 315.

Here £7545·825 ÷ 315 = £23·955 = £23 9f. 5c. 5m.

Ex. 8. Divide 4376 tons 18 cwt. 3 qrs. 14 lbs. by 87.

Taking the decimals for the lower denominations we have 4376·94375 ÷ 87 = 50·309698 tons = 50 tons 6 cwt. 0 qrs. 21 lbs. 11 oz. 9·22 drs.

There is thus obviously a great saving of labour in working questions under the Compound Rules as here shown, and the saving would be vastly greater if we had a decimal system of weights and measures.

23. The present coinage may be readily converted into the new by any of the following methods:—(1.) Reduce the shillings, pence, and farthings to the decimal of £1, and take the first three figures of the decimal. Or (2.) allow 1000 mils to every pound, 100 mils for every pair of shillings, that is, every florin, 50 mils for every shilling over, 25 mils for every sixpence, and a mil for every farthing remaining. Or, more briefly, thus: (3.) Put in the first decimal place half the number of shillings, and in the second place a 5 if there be a shilling over; for 6d. of the pence add 25 mils, and take the farthings in the pence and farthings remaining, as mils.

Thus, to convert 18s. 9d., we have—

$$\begin{array}{rrl}
\text{18s.} = & 900 & \text{mils.} \\
\text{6d.} = & 25 & ,, \\
\text{3d.} = \text{12 far.} = & 12 & ,, \\
\text{18s. 9d.} = & \overline{937} & \text{mils.}
\end{array}
\qquad
\begin{array}{rl}
\text{Or, 18s.} = & £{\cdot}9 \\
\text{6d.} = & \phantom{£{\cdot}}25 \\
\text{3d.} = & \phantom{£{\cdot}}12 \\
\hline
\text{18s. 9d.} = & {\cdot}937 \text{ mils.}
\end{array}$$

This is not, however, quite correct; it is taking the farthings in 3d. as exactly equal to so many mils, as if the pound consisted of 1000 farthings instead of 960. But since 1 farthing = 1$\frac{1}{24}$ mils, 12 farthings = 12½ mils; hence, 18s. 9d. = 937½ mils. = £0·9375. Now, this is the decimal found by reducing 18s. 9d. to the decimal of £1. If, however, it be enacted, in introducing the new coinage, that the penny be reduced in value 4 per cent., in other words, that 25 New Farthings (mils) be given in change for 6d., instead of 24 farthings as at present, the above rule will give the mils exactly, so that 18s. 9d. would be equal to 937 mils. But under the present system the rule does not give the true equivalent for the farthings remaining after the last sixpence or last shilling. The error is, however, so trifling that it may be neglected, as it only amounts to 1 mil when there are 24 farthings.[5]

Ex. 1. Express £11 9s. 9¾d. in the new coinage.

Here, £11 = 11000 mils.
8s. = 400 „
1s. = 50 „
6d. = 25 „
3¾d. = 15 far. = 15 „
11,490 mils = £11·490.

Or, 9s. = ·45
6d. = 25
3¾d. = 15
·490

But since in a pound consisting of 960 farthings, 15 farthings = 15$\frac{15}{24}$ mils = 15⅝ mils, this would be £11·490⅝ = £11·490625.

Ex. 2. Express £5 17s. 7½d. in the new coinage.

12 | 7·500
20 | 17.6250
£5·88125, or £5 8f. 81¼m.

---

[5] The propriety and justice of making the penny the unit has been strongly urged by several able writers on the decimal coinage. It is argued that the farthing should remain; and one proposal is that a new gold coin, to be called a Victoria, equal in value to 1000 farthings, or 10d. more than the sovereign, should be gradually introduced as the sovereigns are returned to the Bank of England; while other coins should be issued of the value of 10 farthings and 100 farthings. This plan would render unnecessary any change in postage and receipt stamps, penny tolls, &c.; by a reduction in the value of the penny, these must either bring 4 per cent. less of revenue than at present, or must be raised to the value of five-mil coins, and so yield 20 per cent. more. In the *Brit. Alm. and Comp.* for 1854, this subject is discussed with that ability and clearness which distinguish all the articles from the same pen. Mr. Theodore W. Rathbone has ably advocated the plan of the penny unit; *Ridgway, Piccadilly*, 1854.

Or thus,
$$\begin{array}{rl} £5\ 17\text{s.} = & £5{\cdot}85 \\ 6\text{d.} = & 25 \\ 1\tfrac{1}{2}\text{d.} = & 6 \\ \hline & £5{\cdot}881 \end{array}$$

EXERCISES.

Express the following in the new coinage, and as decimals of £1 :—

1. 15s. $9\frac{3}{4}$d.
2. 12s. $3\frac{1}{4}$d.
3. 8s. 6d.
4. £3 17s. $10\frac{1}{2}$d.
5. £1 13s. $5\frac{3}{4}$d.
6. £12 14s. 7d.
7. £2 15s. 9d.
8. £7 12s. $11\frac{1}{4}$d.
9. £10 9s. $10\frac{3}{4}$d.
10. 16s. $8\frac{3}{4}$d.
11. £1 19s. $7\frac{1}{2}$d.
12. 13s.

24. The new coinage will be converted into the present by allowing £1 for every 1000 mils; 2s. for every florin or 100 mils; 6d. for every 25 mils; and $\frac{24}{100}$d. for every mil left; or, by using the Rule of Art. 20 for finding the value of a given decimal.

Thus, 4687 mils is £4·687.

Then, reducing ·687 in the usual way, we find for the required value £4 13s. $8\frac{3}{4}{\cdot}\frac{13}{25}$.

$$\begin{array}{r} {\cdot}687 \\ 20 \\ \hline 13{\cdot}74 \\ 12 \\ \hline 8{\cdot}88 \\ 4 \\ \hline 3{\cdot}52 \end{array}$$

Or, otherwise, thus :—

$$\begin{array}{rlrrrl} 4000 & \text{mils} = & £4 & 0 & 0 & \\ 600 & ,, \ = & 0 & 12 & 0 & \\ 25\times3=75 & ,, \ = & 0 & 1 & 6 & \\ 12 & ,, \ = & 0 & 0 & 2\frac{22}{25}, & \text{since } \frac{24\times12}{100}=\frac{288}{100}=2\frac{22}{25}. \\ \hline 4687 & \text{mils} = & £4 & 13 & 8\frac{22}{25}. & \end{array}$$

But $2\frac{22}{25}$d. $= 2\frac{3}{4}{\cdot}\frac{13}{25}$, hence the answer is £4 13s. $8\frac{3}{4}{\cdot}\frac{13}{25}$, the same as by the other method.

The answer will, however, be found with sufficient exactness by setting down as farthings the mils that remain after estimating for the sixpences by taking 25 mils, 50 mils, or 75 mils. Thus, in the above example, for 12 mils put 12 farthings, or 3d., and the answer will be £4 13s. 9d., which is only $\frac{12}{25}$ths of a farthing too great.

Express £7 5f. 6c. 9m. in the present coinage.

Ans. £7 11s. $4\frac{1}{2}\cdot\frac{6}{25}$q.; or £7 11s. $4\frac{3}{4}$d., if 1000 farthings be reckoned to the pound after the last shilling.

Or thus,

£7·5	=	£7	10	0
50 mils	=	0	1	0
19 „	=	0	0	$4\frac{3}{4}$
£7·569		£7	11	$4\frac{3}{4}$

Ex. 1. What is 1376c. 18m. in the present coinage?
Ex. 2. Express 2459m. in pounds, shillings, and pence.
Ex. 3. Express 6675m. in pounds, shillings, and pence.
Ex. 4. Express $18721\frac{7}{8}$m. in pounds, shillings, and pence.
Ex. 5. Express 176fl. 25m. in pounds, shillings, and pence.

25. We shall now further illustrate the advantage of using the new decimal denominations of the pound, and decimals generally, for abridging arithmetical processes, by means of a few examples taken from the rules of Simple Proportion, Practice, Interest, and Discount. Examples have been already given under the Compound Rules.

SIMPLE PROPORTION.

Ex. 1. If £13 14s. $9\frac{3}{4}$d. buy 324 yards, how many yards may be bought for £46 11s. $7\frac{1}{4}$d., at the same rate?

Here, taking from the first and second columns of Table I. the decimal equivalents of the shillings and pence in mils, the statement and work are as on the margin. There is a saving of 58 figures on this mode of working the question, over the ordinary way of reducing the first and second terms to farthings. The answer, however, is only approximately correct, the farthings being taken as mils.

```
As 13·74 : 46·58 : : 324
               324
             -----
             18632
             9316
            13974
      13·74)1509192(1098·34
            1374···
            -----
             13519
             12366
             -----
              11532
              10992
              -----
               5400
               5396
               ----
                 14
```

Ex. 2. If $13\frac{5}{8}$ yards cost £11 9s. $9\frac{3}{4}$d., what will $34\frac{3}{4}$ yards cost at the same rate?

The first ratio being simplified by multiplying the terms by 8, and the decimal for the shillings being taken from the Table, we have the statement—

As 109 : 278 = 11·49 :

and the answer is 29·304, or £29 6s. 1d. (Tables.)

If an additional place be worked for, 7 is found, and the fraction then is $\frac{0}{4}\cdot\frac{1}{2}$. The work is a great saving on the reduction of the shillings, &c., to farthings, the multiplication and division of much larger numbers than are here used, and the reduction of the farthings of the quotient to pounds, shillings, and pence.

Ex. 3. If 12 tons 15 cwt. 3 q. 21 lbs. cost £54 12s. 9¾d. what will be the price of 45 tons 18 cwt. 3 q. 14 lbs. at the same rate?

Here, the lower denominations of the weights being reduced to decimals of the highest, and the mils taken from the Table, the statement is as annexed. The first ratio being reduced, and the operations performed in the usual way, the answer is £196·170 mils, or £196 3s. 5d., in the decimal coinage; but £196 3s. $4\frac{3}{4}\cdot\frac{1}{5}$, if 960 farthings be reckoned to the pound. If the remaining figures of the decimal 2759 be valued, the fraction of a farthing in the answer is $\frac{93}{200}$ instead of $\frac{1}{5}$.

12·796875 : 45·94375 : : 54·640,
or
·20475 : ·7351 : : 54·64 : 196·1702759
Ans. £196 3s. 5d.

Ex. 4. If 35 yds. cost £10 837½ m., what will 49 yds. cost?

Cost of 35 yds. = £10·8375
„ 7 „ = 2·1675
„ 49 „ = 15·1725 = £15 172½m.

Ex. 5. If by selling goods to the value of £324 10s. 7½d. there is a gain of £37 10s. 3¾d., what will be the gain by selling goods to the value of £520 13s. 3¼d.?

The statement is—

As £324 10s. 7½d. : £37 10s. 3¾d. : : £520 13s. 3¼d. :

and the question would be worked in the ordinary way by reducing the terms to farthings, multiplying and dividing, and then reducing the farthings of the quotient to pounds, shillings, and pence. In this way the work would occupy 42 lines, and 225 figures would be employed. But if we take from the third column of the Table the decimal equivalents of the shillings and pence, and use the Contracted Method of multiplying (Art. 16), limiting to five places of decimals, the work will be shortened as follows :—

```
324·53125 : 37·515625 : : 520·6635416̇ :
                          52651573
                         ——————————
                         1561990625
                          364464479
                           26033177
                             520664
                             260331
                              31239
                               1041
                                260
               32453125)1953301816(£60·1884
                ·····  194718750         20
                      —————————       —————
                         611431         3·768
                         324531            12
                       ————————       —————
                         286900         9·216
                         259625           · 4
                       ————————       —————
                          27275         0·864 = 108/125
                          25962
                        ———————
                           1313
                           1298
                         ——————
                             15
```

The answer is thus £60 3s. $9\frac{0}{4}\cdot\frac{216}{250}$d. ; by the ordinary process, the answer comes out £60 3s. $9\frac{0}{4}\cdot\frac{217}{250}$d. The use of the Contracted Method greatly shortens such processes. If the pound of 1000 mils be used, the operation will be much more brief; it will then stand as follows :—

```
£324·531 : 37·515 = £520·663 :
                       51573
                    ——————————
                    15619890
                     3644641
                      260332
                        5207
                        2603
           324531)19532673(£60·187 = £60  3s. 9d.
             ····  1947186
                  ————————
                      6081
                      3245
                    ——————
                      2836
                      2596
                    ——————
                       240
                       227
                     ————
                        13
```

In this operation there are only 84 figures employed. By the ordinary method, after reduction of the first ratio by 5, there are in all 181 figures used, or, with the work for finding the fraction of the farthings, 225 figures.

EXERCISES.

1. If 24 tons 18 cwt. 1 q. 7 lbs. cost £112 14s. 1½d., what will be the price of 60 tons 15 cwt. 2 q. 21 lbs. at the same rate?

2. If £69 7s. 6d. buy 405 yards, how many yards may be bought for £41 12s. 6d. at the same rate?

3. If 312¼ yards cost £92 11s. 10½d., how much will 416⅞ yards cost at the same rate?

4. If £740 14s. 4d. be the rent of 396 ac. 3 r. 10 p., what will be the rent of 516 ac. 2 r. 30 p. at the rame rate?

5. If £24 14s. 7¼d. buy 416 cwt., how many may be bought for £42 17s. 2½d., taking the farthings as mils?

6. If 84 tons 15 cwt. 3 q. cost £317 18s. 1½d., what will 92 tons 10 cwt. 2 q. cost at the same rate?

7. If 2 cwt. 3 q. 21 lbs. cost £12 3s. 4d., what will 17 cwt. 2 q. 14 lbs. cost at the same rate?

8. If by selling goods to the amount of £576 16s. 7¼d. there is a gain of £60 14s. 6d., what would be the gain by selling goods to the value of £720 17s. 9¾d.; the pound sterling being supposed to be divided into 1000 mils?

9. If a ton of wheat cost £8 10s. 0d., what is the cost of the Irish barrel (280 lbs.), of the Liverpool cental (100 lbs.), and of the Glasgow boll (240 lbs.)?

10. If 52·815 centals cost £22·345, what will 192·664 centals cost at the same rate?

11. If 27 cwt. 2 q. 22 lbs. cost £316 14s. 9d., what will be the cost of 48·75 centals, at the same rate?

12. Find a fourth proportional to ·002, ·84, and 168·04.

13. The moon describes an arc of 13°·1764 in 24 h., how much of this will be passed over in 8 h. 20 m. 15 s.?

14. If 324·75 centals of wheat cost £121 12s. 6d., what will be the price of 56 cwt. 3 qrs. 14 lbs. at the same rate?

15. If 124·75 centals cost 464 fl. 25 mils, what will 202·8 centals cost at the same rate?

16. If 100 centals cost £36·475, what will 182·4 centals cost at the same rate?

PRACTICE.

26. By taking all fractions that occur, and all the lower denominations of compound quantities, decimally, questions under this Rule are reduced to that of Simple Multiplication.

Ex. 1. Required the price of 738¾ yds. at £2 16s. 5¼d.

Here, taking the decimals and using the Contracted Method, so as to obtain four decimal places, the answer is found to be £2084 13s. 2¼·⅗d.

If the same question be worked according to the decimal coinage, by taking the mils in 16s. 5¼d. (Table I.), we have the product 738·75 × 2·821 = 2084·01375, which gives £2084 0s. 3¼·⅕d. Here, since the one decimal is ·821, and the other, ·821875 = ·821⅞, ⅞ths of a mil have been rejected; and we must therefore find the price of 738¾ yards at ⅞ths of a mil per yard. Hence, multiplying 738¾ by ⅞, we have 646$\frac{13}{32}$ mils. This will be reduced to farthings by the statement

1$\frac{1}{24}$: 1 : : 646$\frac{13}{32}$ mils : 620$\frac{11}{20}$ farthings.

Now, 620$\frac{11}{20}$ farthings make 12s. 11$\frac{0}{4}$·$\frac{11}{20}$; which being added to the former answer, gives £2084 13s. 2¼·¾d, as before.

$$\begin{array}{r}
2{\cdot}821875 \\
57837 \\
\hline
19753125 \\
846562 \\
225750 \\
19753 \\
1411 \\
\hline
£2084{\cdot}6601 \\
20 \\
\hline
13{\cdot}2 \\
12 \\
\hline
2{\cdot}4 \\
4 \\
\hline
1{\cdot}6 = 1\tfrac{3}{5}\text{q.}
\end{array}$$

£2084	0	3¼·⅕
	12	11$\frac{0}{4}$·$\frac{11}{20}$
£2084	13	2¼·¾

Ex. 2. Find the price of 148 yds. at 36f. 75m. per yard.

Here 3·675 × 148 = 543·900 = £543 18s.

Ex. 3. Required the rent of 564 acres 2 r. 17 p. at £4 7f. 8c. 6m. per acre.

Here, the Contracted Method is used so as to obtain four decimal places, and the decimal is then valued.

$$\begin{array}{r}
564{\cdot}60625 \\
6874 \\
\hline
22584250 \\
3952243 \\
451685 \\
33877 \\
\hline
2702{\cdot}2055
\end{array}$$

or £2702 4s. 1¼·$\frac{7}{25}$.

Ex. 4. Find the price of 215 tons 17 cwt. 3 qrs. 9 lbs. at £9 11s. 6½d. per ton.

Here, the decimal for the lower denominations of the weight is ·891517$\dot{8}57142\dot{}$= ·891517$\frac{6}{7}$, and from Table I. the decimal for the shillings and pence is ·57708$\dot{3}$. Then using the Contracted Method to five places, and rejecting the two last, the answer is £2067 12s. 2½·$\frac{14}{25}$q.

```
215·89151786
  33807759
 ---------
 194302366
  10794576
   1511241
    151124
      1727
        64
         6
-----------
2067·611`04
      20`
    -------
    12·22`
      12`
    -----
     2·64
        4
    -----
     2·56
```

Any other method of working the above question is not to be compared to this for simplicity and brevity. By Practice, many aliquot parts are required, and troublesome fractions arise in the middle of the operation. In the following statement by Proportion,

As 1 ton : 215 tons 17 cwt. 3 q. 9 lbs. : : £9 11s. 6½d. :

as given in Gen. Pasley's evidence before the Committee, p. 26, the work occupies 208 figures; in the foregoing operation only 78. In fact, by the employment of decimals in the price, and for the lower denominations of compound quantities, all questions in the rule of Practice come under Simple Multiplication, and the abridgment of labour in many cases is immense.—When the given quantity is in one denomination and the price can be readily resolved into a few simple aliquot parts, there is little, if any, advantage in the use of decimals.

EXERCISES.

1. 324 at £5.465.
2. 712 at £3 6s. 7½d.
3. 148 at £2 17s. 5¼d.
4. 216 at £3 19s. 9d.
5. 164 at 34 fl. 75 mls.
6. 96 at £1 14s. 0¾d.
7. 144 at £2 15s. 9d.
8. 84 at £3 17s. 3d.
9. 79¾ at £1 2s. 11d.
10. 244 at £4 15s. 2¼d.
11. 112 at 4865 mls.
12. 72⅝ at £3 7 fl. 7 ct. 5 mls.
13. 695 at 14s. 10d.
14. 48 at 56 fl. 15 mls.
15. 64 at 16s. 11¼d.
16. 3156 at £2 15s. 11¾d.
17. 116 at £1 13s. 7¼d.
18. 3749⅜ at £3 15s. 6d.
19. 1375$\frac{5}{9}$ at £2 19s. 11¾d.
20. 574⅗ at £19 11s. 6d.
21. 42 cwt. 3 q. 14 lbs. at £2 7s. 9d. per cwt.

22. 212 cwt. 3 q. 19 lbs. at £1 13s. 2d. per cwt.
23. 75 cwts. 3 q. 21 lbs. at £4 14s. 6d. per cwt.
24. 92 tons 18 cwt. 1 q. 14 lbs. at £10 16s. 10½d. per ton.
25. 647 cwt. 2 q. 11 lbs. at £6 10s. 8d. per cwt.
26. 512 cwt. 3 q. 7 lbs. at 7248 mls. per cwt.
27. 14 cwt. 3 q. 21 lbs. at 9s. 6d. per cental.
28. 234·75 centals at 8s. 6d. per cental.
29. 356·86 centals at 12s. 6d. per cwt.
30. 10 tons at 8s. 7½d. per cental.
31. 4 tons 5 cwt. 2 q. 24 lbs. at 5s. 6d. per cental.
32. 34 ac. 2 r. 20 p. at £4 15s. 7½d. per acre.
33. 464 ac. 3 r. 30 p. at 4895 mls. per acre.
34. 139 ac. 3 r. 39 p. at £2 7s. 10d. per acre.
35. 124 oz. 15 dwts. 12 gr. at 3896 mls. per oz.

INTEREST AND DISCOUNT.

27. The ordinary operations in Interest and Discount are embraced under four rules, according as the given time is one year, any number of years, any number of months, or any number of days. But if the given time be always expressed in years or decimals of a year, these four rules will be embraced in this one:

Multiply together continually the principal, rate, and time (in years), and divide the product by 100;

and hence, if the shillings and pence, and fractional rates be also expressed decimally, the division will consist merely in shifting the decimal point two places. Nothing can be more simple and ready than the common operation for one year, since troublesome fractions never arise with any of the usual rates; but in the case of months, years and months, and days, the advantage in most cases is very great by using the above rule. Further, since the order in which the factors are taken does not affect a product, we may take the quantities *principal, rate, time,* and $\frac{1}{100}$, in any order we please, as particular examples may suggest to be most convenient. The decimal, which any given number of days is of a year, will be found in Table III.

Ex. 1. Required the interest of £317 9s. 6d. for 4 years at $3\frac{3}{4}$ per cent.

Here, the product, 15, of the time and rate is first found, and the principal, divided by 100, is then multiplied by this product; and the decimal being valued gives for answer £47 12s. $5\frac{0}{4}\cdot\frac{2}{5}$d. The multiplication by 15 may be shortened as under:

$$\begin{array}{r} 3{\cdot}17475 \\ 1587375 \\ \hline 47{\cdot}62125 \end{array}$$

$$\begin{array}{r} 3{\cdot}17475 \\ 15 \\ \hline 1587375 \\ 317475 \\ \hline 47{\cdot}62125 \\ 20 \\ \hline 12{\cdot}425 \\ 12 \\ \hline 5{\cdot}1 \end{array}$$

Ex. 2. Find the interest of £637 11s. $9\frac{3}{4}$d. for 2 years at $5\frac{1}{4}$ per cent.

The answer will not be affected in this case to the amount of half a farthing, by neglecting the last three figures (625) of the decimal found in Table I., the remainder being only ·302 when these figures are employed. Answer, £66 18s. $11\frac{1}{4}$d.: remainder ·072 farth.

$$\begin{array}{r} 6{\cdot}3759 \\ 10{\cdot}5 \\ \hline 318795 \\ 63759 \\ \hline 66{\cdot}94695 \\ 20 \\ \hline 18{\cdot}939 \\ 12 \\ \hline 11{\cdot}268 \end{array}$$

Ex. 3. Required the discount on a bill of £845 for 73 days at $4\frac{1}{2}$ per cent.

Taking the decimal for 73 days, in Table III., we have 4·5 × ·2 = ·9, and the answer is readily found to be £7 12s. 1d.

$$\begin{array}{r} 8{\cdot}45 \\ {\cdot}9 \\ \hline 7{\cdot}605 \\ 20 \\ \hline 12{\cdot}1 \\ 12 \\ \hline 1{\cdot}2 \end{array}$$

Ex. 4. Required the discount on £174 12s. 6d. for 115 days at 4 per cent.

The decimal for 115 days may here be limited to three figures, as the next is a cipher: then ·315 × 4 = 1·26. The second partial product is obviously got by doubling the first. The discount is £2 4s., with a remainder of ·244 farth.

$$\begin{array}{r} 1{\cdot}74625 \\ 1{\cdot}26 \\ \hline 1047750 \\ 2095500 \\ \hline 2{\cdot}200275 \\ 20 \\ \hline 4{\cdot}0055 \end{array}$$

Ex. 5. Required the interest of £236 16s. $7\frac{3}{4}$d. for 199 days at 5 per cent.

Here, the decimal of a year is ·5452, the next figure being cipher, and in the decimal equivalent of the shillings and pence 3 is put for the last four figures; then using the Contracted Method to obtain four places, the work has the extremely neat and compact form given on the margin. If the operation by any of the common rules be compared with this, the advantage will be seen to be greatly on the side of the decimal method. The decimal equivalent of the shillings and pence, in mils, being ·832, the operation will be exactly the same as the above. The decimal ·456 is then mils, and turning to Table I., we find opposite to this 9s. $1\frac{1}{2}$d.; or, without reference to the Table, we see at once that 4fl. = 8s., 50m. = 1s., 6m. = 6 farthings = $1\frac{1}{2}$d.; the answer in the decimal coinage differing from the former by only $\frac{6}{25}$ farth.

$$\begin{array}{r}
\cdot5452 \times 5 = 2\cdot726 \\
£236\cdot832291\dot{6} \\
\\
£2\cdot368323 \\
6272 \\
\hline
47366 \\
16578 \\
474 \\
142 \\
\hline
6\cdot456 \\
20 \\
\hline
9\cdot12 \\
12 \\
\hline
1\cdot44 \\
4 \\
\hline
1\cdot76 \\
£6\ 9s.\ 1\frac{1}{4}\cdot\frac{19}{25}d.
\end{array}$$

Ex. 6. Required the present worth of a bill of 5346fl. 48m. drawn on the 1st of May at 3 months, discounted May 17, at $6\frac{1}{2}$ per cent.

The term of the bill is 79 days = ·216438 yr., the product of which, by the rate, is 1·406847. The discount is £7 10s. $5\frac{1}{4}$d. Then 648 mils = 12s. $11\frac{3}{4}$d.; and, therefore, the present worth = £527 2s. $6\frac{1}{2}$d. If the mils be estimated in the present coinage, we shall have for present worth, £527 2s. $6\frac{1}{4}\cdot\frac{23}{25}$.

$$\begin{array}{lr}
 & £534\cdot648 \\
 & \\
 & 5\cdot34648 \\
 & 486041 \\
\hline
 & 53465 \\
 & 21386 \\
 & 320 \\
 & 43 \\
 & 2 \\
\hline
 & 7\cdot521|6 \\
\text{Amount,......} & 534\cdot648 \\
\text{Discount,.........} & 7\cdot521 \\
\hline
\text{Pre. worth,...} & 527\cdot127
\end{array}$$

But as discounts are never reckoned to farthings, the answers will be the same in both coinages.

The following example, given by Prof. De Morgan (*Brit. Alm. and Comp.*, 1848), illustrates the immense advantage of the decimal system in all calculations in the "Per-centage Rules." A reference to our first Table still further abbreviates the process as there given.

Ex. 7. How much in the pound is £43 17s. 4$\frac{3}{4}$d. per cent.?

The statement and work are as on the margin; and the answer is at once found, by reference to Table I., to be 8s. 9$\frac{1}{4}$d. If the whole decimal be valued in the usual way, there is a remainder of ·1424 farth.

100 : £43 17s. 4$\frac{3}{4}$d. : : 1 :
100 : £43·869 : : 1 :
rate per £1 = ·43869
= 8s. 9$\frac{1}{4}$d.

Ex. 8. How much in the pound is £2$\frac{11}{12}$ per cent.?

Here £2$\frac{11}{12}$ = £2 18s. 4d. = 2·91$\dot{6}$; hence, the rate per £1 = ·02916, opposite to which, in either the second or third column of Table I., we find 7d., the rate required.

EXERCISES.

1. Interest of £665, for 73 days, at 5 per cent.
2. Interest of £550, for 146 days, at 3 per cent.
3. Discount on £875, for 73 days, at 6 per cent.
4. Discount on £450, for 73 days, at 5 per cent.
5. Interest of £650, for 146 days, at 5 per cent.
6. Interest of £840, for 146 days, at 5 per cent.
7. Interest of £250, for 292 days, at 6 per cent.
8. Present worth of £825, for 292 days, at 5 per cent.
9. Present worth of £450, for 182$\frac{1}{2}$ days, at 4 per cent.
10. Interest of £375, for 73 days, at 4 per cent.
11. Interest of £225, for 292 days, at 4 per cent.
12. Interest of £850, for 146 days, at 5 per cent.
13. Interest of £825, for 219 days, at 6 per cent.
14. Interest of £460, for 182$\frac{1}{2}$ days, at 2 per cent.
15. Discount on £450, for 73 days, at 4 per cent.
16. Present worth of £350, for 219 days, at 3 per cent.
17. Present worth of £555, for 292 days, at 5 per cent.
18. Interest of £888, for 67 days, at 4 per cent.
19. Interest of £234, for 26 days, at 3 per cent.
20. Interest of £520, for 150 days, at 6 per cent.
21. Interest of £317, for 42 days, at 4 per cent.
22. Discount on £215 7s. 6d., for 168 days, at 3 per cent.
23. Present worth of a bill of £364 12s. 6d., for 117 days, at 5 per cent.
24. Discount on a bill of £156 9s. 6d., for 46 days, at 5 per cent.
25. Discount on a bill of £360 15s. 0d., for 188 days, at 4 per cent.

26. Present worth of £712, 765 mils, for 325 days, at 4 per cent.

27. How much in the pound is 33⅞ per cent?

28. An insolvent's estate yields 25⅝ per cent.; what will be the dividend per pound sterling?

29. How much per pound is 3⅛ per cent.?

30. Interest of £647 13s. 9d., for 146 days, at 4 per cent.

31. Interest of £445 11s. 6d., for 219 days, at 2 per cent.

32. Discount on £125 8s. 9d., for 292 days, at 5 per cent.

33. How much in the pound is £34 18s. 8½d. per cent.?

34. Interest of £512 16s. 3d., for 200 days, at 4 per cent.

35. Present worth of a bill of 64576 fl. 56 mils, drawn 1st July, at 3 months, discounted 10th August, at 4 per cent.

36. An insolvent's estate pays £16 11s. 9¾d. per cent.; what will be the dividend per pound sterling?

ANSWERS TO THE EXERCISES.

Art. 6—Page 4.

1. $\frac{3}{1000}$. 2. $\frac{27}{10000}$. 3. ·07. 4. ·00371.
5. ·507. 6. ·0311. 7. ·009027. 8. 15473·6.
9. 184250. 10. 1·1. 11. 57·96. 12. ·02.
13. ·12875. 14. ·00001. 15. 8·756625.

Art. 11—Page 9.

1. ·5, $\cdot\dot{3}$, ·25, ·2, $\cdot 1\dot{6}$, $\cdot\dot{1}4285\dot{7}$, ·125, $\cdot\dot{1}$, $\cdot\dot{0}\dot{9}$, $\cdot 08\dot{3}$.
2. $\cdot\dot{6}$, ·75. ·8, $\cdot 8\dot{3}$, ·375, ·625, ·875, $\cdot\dot{9}\dot{0}$, $\cdot 91\dot{6}$.
3. ·0625, ·1875, ·3125, ·4375, ·5625, ·6875, ·8125, ·9375.
4. ·078125. 5. ·0048828125.
6. ·0078125. 7. ·34375.
8. ·05859375. 9. ·0693359375.
10. ·995. 11. ·9875.
12. ·96. 13. ·984.
14. ·996. 15. ·2512.
16. ·15625. 17. ·48.
18. ·79375. 19. $\cdot\dot{0}1234567\dot{9}$.
20. $\cdot 9\dot{3}$. 21. $\cdot\dot{0}\dot{1}$, $\cdot\dot{0}0\dot{1}$, $\cdot\dot{0}00\dot{1}$.
22. $\cdot\dot{1}32075471698\dot{1}$. 23. $\cdot\dot{0}005999\dot{4}$.
24. $\cdot\dot{9}41176470588235\dot{2}$. 25. $\cdot\dot{9}4736842105263157\dot{8}$.
26. $\cdot\dot{4}68354430379\dot{7}$. 27. $\cdot 12\dot{3}4\dot{5}$.
28. $2\cdot\dot{7}85714\dot{2}$. 29. $\cdot\dot{0}7692\dot{3}$, $\cdot 0\dot{7}1428\dot{5}$.
30. $\cdot\dot{0}20,408,163,265,306,122,448,979,591,836,734,693,877,55\dot{1}$.

Art. 12—Page 11.

1. ·95. 2. ·475.
3. ·29375. 4. ·790625.
5. ·00125. 6. ·84.
7. ·81875. 8. ·165625.
9. $\cdot4097\dot{2}$. 10. $\cdot0458\dot{3}$.
11. ·0390625. 12. $\cdot232\dot{5}\dot{7}$.
13. ·0015625. 14. ·953125.
15. ·6. 16. ·7359375.
17. $\cdot\dot{7}$. 18. ·384.
19. $\cdot40246\dot{2}\dot{1}$. 20. ·953125.
21. $\cdot82\dot{2}8571\dot{4}$. 22. $\cdot2\dot{3}$.
23. $\cdot2422\dot{4}0\dot{7}$. 24. ·5125.
25. $\cdot911458\dot{3}$.

Art. 13—Page 14.

1. $\frac{7}{9}$. 2. $\frac{13}{90}$. 3. $\frac{5}{36}$. 4. $\frac{4}{27}$.
5. $\frac{1}{72}$. 6. $\frac{3958}{8325}$. 7. $\frac{1}{2}$. 8. $\frac{133}{3000}$.
9. $\frac{145}{222}$. 10. $\frac{4}{7}$. 11. $\frac{9}{14}$. 12. $\frac{25}{44}$.

Art. 14—Page 15.

1. 35·837034. 2. 913·050725.
3. 127·37512045. 4. 534·105785.
5. 3·28295175. 6. 24·02628.
7. 65·504414. 8. 105·351152.
9. $3\cdot90\dot{7}1428\dot{5}$. 10. $2\cdot18670\dot{0}23281607\dot{4}\dot{4}$.
11. $7\cdot55\dot{8}\dot{2}$. 12. ·9375.
13. 4·219576. 14. ·009.
15. 254·735915. 16. $6\cdot42453\dot{4}\dot{1}$.
17. $11\cdot946\dot{4}28571\dot{1}$. 18. $\cdot3882\dot{7}02156\dot{6}$.
19. $5\cdot28\dot{2}2\dot{9}$. 20. $\cdot458\dot{8}56037\dot{7}$.
21. $\cdot\dot{4}7210\dot{8}$. 22. $\cdot254\dot{8}566375132698221671\dot{6}$.

ART. 15—PAGES 17, 18.

1. 99·693.
2. 12·996
3. 9·92.
4. ·022308.
5. 21·3978.
6. 12.
7. 180.
8. ·8.
9. 4843·000783125.
10. 1·2321.
11. 2736·328125.
12. ·0441.
13. 2·419003.
14. ·7777.
15. 4027·5265.
16. ·1740324.
17. $\cdot 6408\dot{3}\dot{9}$.
18. $599\cdot 6272\dot{0}7\dot{7}$.
19. $\cdot 27774\dot{2}59\dot{6}$.
20. $40\cdot 921\dot{2}0\dot{9}$.
21. $24\cdot 63\dot{1}$.
22. 8·2875.
23. ·4096.
24. ·001478741.
25. 1·5.

ART. 17—PAGE 21.

1. 54·625.
2. 3·72714.
3. 267·706650.
4. 1·686659.
5. ·34028.
6. 4·8221650.
7. ·12689.
8. 2·45975.

ART. 18—PAGE 24.

1. 5·128.
2. 34·005.
3. ·326.
4. 1·1.
5. 10.
6. ·37.
7. ·125.
8. 1240.
9. 750.
10. 100.
11. ·00007.
12. 460.
13. ·01.
14. 2500.
15. 1170.
16. 3·21685.
17. 856·3192.
18. 14·24611.
19. 1·379675.
20. $\cdot\dot{4}8\dot{1}$.
21. $2\cdot 280283133640552\frac{316}{247}$; ·318309 . . . ; $\cdot 7\dot{3}5$.
22. ·001 ; 10 ; 10,500 ; 1000.
23. 18·75 ; ·8204 ; 185 700 000.
24. 2670 ; 267 ; 26·7 ; 2·67.
25. 36·48 ; 3·648 ; ·3648 ; ·03648.

Art. 19—Page 26.

1. 1·1715. 2. 4·72976. 3. ·04549495.

4. $\cdot\dot{3}\dot{9}$. 5. 758·8863. 6. 110·64933.

Art. 20—Page 28.

1. 15s. 4½d.
2. 7s. 8¾d.
3. 19s. $5\frac{1}{2}\cdot\frac{1}{3125}$q.
4. 8 sq. poles.
5. 10 sq. poles.
6. 1 r. 2 p. 23·716 sq. yds.
7. 7s. 10½d.
8. 4s. $11\frac{3}{4}\cdot\frac{2}{5}$.
9. 2 qrs. 20 lb. 10 oz. 9·984 dr.
10. 3 cwt. 1 qr. 7 lbs.
11. 10 oz. ·6 grs.
12. 10s. 1½d.
13. 4s. 2d.
14. 1s. 9½d.
15. 3 qrs.
16. 3 m. 56·6496 s.
17. 59′ 8·34″.
18. 2′
19. 1s. 8d.
20. 16 lbs.
21. 14 lbs. 3 oz. 2 grs.
22. 1 qr. 4 lbs.
23. 9s.
24. 3 f. 10 p. 3 yds. 2 ft.
25. 13 d. 17 h. 1 m. $32\frac{4}{13}$ s.

Art. 25—Page 38.

Several of these exercises are intended to illustrate the application of the contracted methods of multiplying and dividing. The work of the first is given at length.

$$\begin{array}{r}
60{\cdot}784375 \\
52607211 \\
\hline
60784375 \\
6078438 \\
1215687 \\
425491 \\
3647 \\
121 \\
30 \\
\hline
\end{array}$$

1. Here the answer is required to be true to four decimal places, and the unit figure of the multiplier is placed accordingly under the fourth

decimal figure of the multiplicand. The contracted method of division is then employed, the divisor being placed to the right of the dividend, for greater convenience in multiplying. The answer is thus found to be £274 19s. $2\frac{0}{4}$·736. The answer found by the uncontracted process, in which about one hundred figures *more* are employed, differs only in the second decimal place of the fraction remaining.

6850·7789	24·915625
49831250	274·9591
18676539	20
1744093$\dot{7}$	19·1820
1235602	12
996625	2·184
238977	4
224240	0·736
14737	
12458	
2279	
2242	
37	
25	
12	

2. 243 yds.
3. £123 12s. $4\frac{1}{2}$d.
4. £964 9s. $7\frac{3}{4}$d.
5. 720 cwt. 3 qrs. 25 lbs.
6. £346 18s. 4d.
7. £73.
8. £75 890 mls.; or £75 17s. $9\frac{3}{4}$d.
9. £1 1s. 3d.; 7s. $7\frac{0}{4}\cdot\frac{2}{7}$; 18s. $2\frac{1}{2}\cdot\frac{2}{7}$q.
10. £81 10s. $2\frac{3}{4}$d.
11. £497 15s. $5\frac{3}{4}$d.
12. 70576·8.
13. 4° 34′ 38″.
14. £23 16s. $1\frac{1}{2}$d.
15. £75 9s. 5d.
16. £66 10s. $7\frac{1}{4}$d.

ART. 26—PAGE 39.

1. £1770 13s. $2\frac{1}{4}\cdot\frac{3}{5}$.
2. £2371 17s. 0d.
3. £425 0s. 9d.
4. £861 6s. 0d.
5. £569 18s. 0d.
6. £163 10s. 0d.
7. £401 8s. 0d.
8. £32 9s. 0d.
9. £91 7s. $7\frac{0}{4}$·958.
10. £1161 5s. 9d.
11. £544 17s. $7\frac{0}{4}\cdot\frac{4}{5}$.
12. £274 3s. $2\frac{1}{4}$d.
13. £515 9s. 2d.
14. £269 10s. $4\frac{3}{4}\cdot\frac{1}{5}$.
15. £54 4s. 0d.
16. £8833 10s. 3d.
17. £194 18s. $0\frac{3}{4}$·996.
18. £14153 17s. $9\frac{3}{4}$d.
19. £4125 4s. $8\frac{0}{4}\cdot\frac{4}{5}$.
20. £11247 15s. $10\frac{2}{4}\cdot\frac{1}{5}$.

21. £102 7s. $3\frac{1}{4}\cdot\frac{1}{3}$.
22. £353 1s. 10d.
23. £358 16s. $1\frac{0}{4}\cdot\frac{1}{3}$.
24. £1007 11s. $9\frac{0}{4}\cdot\frac{3}{16}$.
25. £4230 19s. $5\frac{1}{4}\cdot 9$.
26. £3716 17s. $3\frac{1}{4}\cdot\frac{3}{8}$q.
27. £7 18s. $11\frac{0}{4}\cdot 88$.
28. £99 15s. $4\frac{1}{2}$d.
29. £199 2s. $9\frac{3}{4}$d.
30. £96 12s. 0d.
31. £506 8s. 0d.

ART. 27—PAGE 44.

1. £6 13s. 0d.
2. £6 12s. 0d.
3. £10 10s. 0d.
4. £4 10s. 0d.
5. £13 0s. 0d.
6. £16 16s. 0d.
7. £12 0s. 0d.
8. £792 0s. 0d.
9. £441 0s. 0d.
10. £3 0s. 0d.
11. £7 4s. 0d.
12. £17 0s. 0d.
13. £29 14s. 0d.
14. £4 12s. 0d.
15. £3 12s. 0d.
16. £343 14s. 0d.
17. £532 16s. 0d.
18. £6 10s. $4\frac{3}{4}\cdot 28$.
19. £0 10s. 0d.
20. £12 16s. 5d.
21. £1 9s. 2d.
22. £2 19s. $5\frac{1}{4}$d.
23. £358 15s. 10d.
24. £0 19s. $8\frac{1}{2}$d.
25. £7 8s. $7\frac{1}{2}$d.
26. £25 7s. $5\frac{3}{4}$d.
27. £0 6s. $9\frac{1}{4}\cdot\frac{1}{5}$.
28. £0 5s. $1\frac{1}{2}$d.
29. £0 0s. $7\frac{1}{2}$d.
30. £10 7s. $3\frac{0}{4}\cdot\frac{12}{25}$.
31. £5 6s. $11\frac{1}{4}\cdot 624$.
32. £5 0s. $4\frac{0}{4}\cdot\frac{4}{5}$.
33. £0 6s. $11\frac{3}{4}\cdot 38$.
34. £11 4s. $9\frac{1}{2}\cdot 133$q.
35. 64189 fl. 10 mls.
36. £0 3s. $3\frac{3}{4}\cdot\frac{1}{5}$

EXPLANATION OF THE TABLES.

The following Tables have been prepared for the purpose of shortening the processes of calculation recommended and illustrated in the second part of this tract. They will be found equally applicable whether the present or proposed new denominations of money be employed. The first Table gives the decimal equivalents of the various subdivisions of the present pound sterling; and also these same subdivisions expressed in mils, or thousandths of the present pound. As far as 4d., these subdivisions extend to sixteenths of a penny, fractions often required in mercantile calculations; from 4d. to 6d., they extend as far as eighths; and from 6d. upwards, to farthings only. The computation has been made by the Rules of Arts. 12, 23. This Table is also useful in finding, by inspection, the value of any given decimal of a pound sterling, to the nearest farthing. The second Table is formed by successive multiplication, from the value of one mil, given in Art. 22. The third Table has been computed by the Rule of Art. 12. The decimals are carried out to six places, in order to insure accuracy in the fourth place; as, for most purposes, it is quite unnecessary to use any figure beyond that place.

TABLE I.

Equivalents of Shillings, Pence, and Farthings in Decimals of the present Pound of 960 Farthings, and also of the Pound of 1000 Mils of the Proposed New Coinage.

s.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	D.	Decimals of £1 in Mils.	Decimals of the present £1.
0	$0\frac{1}{16}$	·00025	·000260416′	0	$2\frac{5}{8}$	·0105	·0109375		$6\frac{3}{4}$	·028	·028125
	$0\frac{1}{8}$	·0005	·00052083′		$2\frac{11}{16}$	·01075	·011197916′	0	7	·029	·02916′
	$0\frac{3}{16}$	·00075	·00078125		$2\frac{3}{4}$	·011	·0114583′		$7\frac{1}{4}$	·030	·0302083′
	$0\frac{1}{4}$	·001	·0010416′		$2\frac{13}{16}$	·01125	·01171875		$7\frac{1}{2}$	·031	·03125
	$0\frac{5}{16}$	·00125	·001302083′		$2\frac{7}{8}$	·0115	·01197916′		$7\frac{3}{4}$	·032	·0322916′
	$0\frac{3}{8}$	·0015	·0015625		$2\frac{15}{16}$	·01175	·012239583′	0	8	·033	·03′
	$0\frac{7}{16}$	·00175	·001822916′	0	3	·012	·0125		$8\frac{1}{4}$	·034	·034375
	$0\frac{1}{2}$	·002	·002083′		$3\frac{1}{16}$	·01225	·012760416′		$8\frac{1}{2}$	·035	·035416′
	$0\frac{9}{16}$	·00225	·00234375		$3\frac{1}{8}$	·0125	·01302083′		$8\frac{3}{4}$	·036	·0364583′
	$0\frac{5}{8}$	·0025	·00260416′		$3\frac{3}{16}$	·01275	·01328125	0	9	·037	·0375
	$0\frac{11}{16}$	·00275	·002864583′		$3\frac{1}{4}$	·013	·0135416′		$9\frac{1}{4}$	·038	·0385416′
	$0\frac{3}{4}$	·003	·003125		$3\frac{5}{16}$	·01325	·013802083′		$9\frac{1}{2}$	·039	·039583′
	$0\frac{13}{16}$	·00325	·003385416′		$3\frac{3}{8}$	·0135	·0140625		$9\frac{3}{4}$	·040	·040625
	$0\frac{7}{8}$	·0035	·00364583′		$3\frac{7}{16}$	·01375	·014322916′	0	10	·041	·0416′
	$0\frac{15}{16}$	·00375	·00390625		$3\frac{1}{2}$	·014	·014583′		$10\frac{1}{4}$	·042	·0427083′
0	1	·004	·00416′		$3\frac{9}{16}$	·01425	·01484375		$10\frac{1}{2}$	·043	·04375
	$1\frac{1}{16}$	·00425	·004427083′		$3\frac{5}{8}$	·0145	·01510416′		$10\frac{3}{4}$	·044	·0447916′
	$1\frac{1}{8}$	·0045	·0046875		$3\frac{11}{16}$	·01475	·015364583′	0	11	·045	·04583′
	$1\frac{3}{16}$	·00475	·004947916′		$3\frac{3}{4}$	·015	·015625		$11\frac{1}{4}$	·046	·046875
	$1\frac{1}{4}$	·005	·0052083′		$3\frac{13}{16}$	·01525	·015885416′		$11\frac{1}{2}$	·047	·047916′
	$1\frac{5}{16}$	·00525	·00546875		$3\frac{7}{8}$	·0155	·01614583′		$11\frac{3}{4}$	·048	·0489583′
	$1\frac{3}{8}$	·0055	·00572916′		$3\frac{15}{16}$	·01575	·01640625	1	0	·050	·050
	$1\frac{7}{16}$	·00575	·005989583′	0	4	·016	·016′		$0\frac{1}{4}$	·051	·0510416′
	$1\frac{1}{2}$	·006	·00625		$4\frac{1}{8}$	·0165	·0171875		$0\frac{1}{2}$	·052	·052083′
	$1\frac{9}{16}$	·00625	·006510416′		$4\frac{1}{4}$	·017	·0177083′		$0\frac{3}{4}$	·053	·053125
	$1\frac{5}{8}$	·0065	·00677083′		$4\frac{3}{8}$	·0175	·01822916′	1	1	·054	·05416′
	$1\frac{11}{16}$	·00675	·00703125		$4\frac{1}{2}$	·018	·01875		$1\frac{1}{4}$	·055	·0552083′
	$1\frac{3}{4}$	·007	·0072916′		$4\frac{5}{8}$	·0185	·01927083′		$1\frac{1}{2}$	·056	·05625
	$1\frac{13}{16}$	·00725	·007552083′		$4\frac{3}{4}$	·019	·0197916′		$1\frac{3}{4}$	·057	·0572916′
	$1\frac{7}{8}$	·0075	·0078125		$4\frac{7}{8}$	·0195	·0203125	1	2	·058	·0583′
	$1\frac{15}{16}$	·00775	·008072916′	0	5	·020	·02083′		$2\frac{1}{4}$	·059	·059375
0	2	·008	·0083′		$5\frac{1}{8}$	·0205	·02135416′		$2\frac{1}{2}$	·060	·060416′
	$2\frac{1}{16}$	·00825	·00859375		$5\frac{1}{4}$	·021	·021875		$2\frac{3}{4}$	·061	·0614583′
	$2\frac{1}{8}$	·0085	·00885416′		$5\frac{3}{8}$	·0215	·02239583′	1	3	·062	·0625
	$2\frac{3}{16}$	·00875	·009114583′		$5\frac{1}{2}$	·022	·022916′		$3\frac{1}{4}$	·063	·0635416′
	$2\frac{1}{4}$	·009	·009375		$5\frac{5}{8}$	·0225	·0234375		$3\frac{1}{2}$	·064	·064583′
	$2\frac{5}{16}$	·00925	·009635416′		$5\frac{3}{4}$	·023	·0239583′		$3\frac{3}{4}$	·065	·065625
	$2\frac{3}{8}$	·0095	·00989583′		$5\frac{7}{8}$	·0235	·02447916′	1	4	·066	·06′
	$2\frac{7}{16}$	·00975	·01015625	0	6	·025	·025		$4\frac{1}{4}$	·067	·0677083′
	$2\frac{1}{2}$	·010	·010416′		$6\frac{1}{4}$	·026	·0260416′		$4\frac{1}{2}$	·068	·06875
	$2\frac{9}{16}$	·01025	·010677083′		$6\frac{1}{2}$	·027	·027083′		$4\frac{3}{4}$	·069	·0697916′

s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.
1	5	·070	·07083′	2	5	·120	·12083′	3	5	·170	·17083′
	5¼	·071	·071875		5¼	·121	·121875		5¼	·171	·171875
	5½	·072	·072916′		5½	·122	·122916′		5½	·172	·172916′
	5¾	·073	·0739083′		5¾	·123	·1239083′		5¾	·173	·1739583′
1	6	·075	·075	2	6	·125	·125	3	6	·175	·175
	6¼	·076	·0760416′		6¼	·126	·1260416′		6¼	·176	·1760416′
	6½	·077	·077083′		6½	·127	·127083′		6½	·177	·177083′
	6¾	·078	·078125		6¾	·128	·128125		6¾	·178	·178125
1	7	·079	·07916′	2	7	·129	·12916′	3	7	·179	·17916′
	7¼	·080	·0802083′		7¼	·130	·1302083′		7¼	·180	·1802083′
	7½	·081	·08125		7½	·131	·13125		7½	·181	·18125
	7¾	·082	·0822916′		7¾	·132	·1322916′		7¾	·182	·1822916′
1	8	·083	·083′	2	8	·133	·13′	3	8	·183	·183′
	8¼	·084	·084375		8¼	·134	·134375		8¼	·184	·184375
	8½	·085	·085416′		8½	·135	·135416′		8½	·185	·185416′
	8¾	·086	·0864583′		8¾	·136	·1364583′		8¾	·186	·1864583′
1	9	·087	·0875	2	9	·137	·1375	3	9	·187	·1875
	9¼	·088	·0885416′		9¼	·138	·1385416′		9¼	.188	·1885416′
	9½	·089	·089583′		9½	·139	·139583′		9½	·189	·189583′
	9¾	·090	·090625		9¾	·140	·140625		9¾	·190	·190625
1	10	·091	·0916′	2	10	·141	·1416′	3	10	·191	·1916′
	10¼	·092	·0927083′		10¼	·142	·1427083′		10¼	·192	·1927083′
	10½	·093	·09375		10½	·143	·14375		10½	·193	·19375
	10¾	·094	·0947916′		10¾	·144	·1447916′		10¾	·194	·1947916′
1	11	·095	·09583′	2	11	·145	·14583′	3	11	·195	·19583′
	11¼	·096	·096875		11¼	·146	·146875		11¼	·196	·196875
	11½	·097	·097916′		11½	·147	·147916′		11½	·197	·197916′
	11¾	·098	·0989583′		11¾	·148	·1489583′		11¾	·198	·1989583′
2	0	·100	·100	3	0	·150	·150	4	0	·200	·200
	0¼	·101	·1010416′		0¼	·151	·1510416′		0¼	·201	·2010416′
	0½	·102	·102083′		0½	·152	·152083′		0½	·202	·202083′
	0¾	·103	·103125		0¾	·153	·153125		0¾	·203	·203125
2	1	·104	·10416′	3	1	·154	·15416′	4	1	·204	·20416′
	1¼	·105	·1052083′		1¼	·155	·1552083′		1¼	·205	·2052083′
	1½	·106	·10625		1½	·156	·15625		1½	·206	·20625
	1¾	·107	·1072916′		1¾	·157	·1572916′		1¾	·207	·2072916′
2	2	·108	·1083′	3	2	·158	·1583′	4	2	·208	·2083′
	2¼	·109	·109375		2¼	·159	·159375		2¼	·209	·209375
	2½	·110	·110416′		2½	·160	·160416′		2½	·210	·210416′
	2¾	·111	·1114583′		2¾	·161	·1614583′		2¾	·211	·2114583′
2	3	·112	·1125	3	3	·162	·1625	4	3	·212	·2125
	3¼	·113	·1135416′		3¼	·163	·1635416′		3¼	·213	·2135416′
	3½	·114	·114583′		3½	·164	·164583′		3½	·214	·214583′
	3¾	·115	·115625		3¾	·165	·165625		3¾	·215	·215625
2	4	·116	·116′	3	4	·166	·16′	4	4	·216	·216′
	4¼	·117	·1177083′		4¼	·167	·1677083′		4¼	·217	·2177083′
	4½	·118	·11875		4½	·168	·16875		4½	·218	·21875
	4¾	·119	·1197916′		4¾	·169	·1697916′		4¾	·219	·2197916′

s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.
4	5	·220	·22083′	5	5	·270	·27083′	6	5	·320	·32083′
	5¼	·221	·221875		5¼	·271	·271875		5¼	·321	·321875
	5½	·222	·222916′		5½	·272	·272916′		5½	·322	·322916′
	5¾	·223	·2239583′		5¾	·273	·2739583′		5¾	·323	·3239583′
4	6	·225	·225	5	6	·275	·275	6	6	·325	·325
	6¼	·226	·2260416′		6¼	·276	·2760416′		6¼	·326	·3260416′
	6½	·227	·227083′		6½	·277	·277083′		6½	·327	·327083′
	6¾	·228	·228125		6¾	·278	·278125		6¾	·328	·328125
4	7	·229	·22916′	5	7	·279	·27916′	6	7	·329	·32916′
	7¼	·230	·2302083′		7¼	·280	·2802083′		7¼	·330	·3302083′
	7½	·231	·23125		7½	·281	·28125		7½	·331	·33125
	7¾	·232	·2322916′		7¾	·282	·2822916′		7¾	·332	·3322916′
4	8	·233	·23′	5	8	·283	·283′	6	8	·333	·3′
	8¼	·234	·234375		8¼	·284	·284375		8¼	·334	·334375
	8½	·235	·235416′		8½	·285	·285416′		8½	·335	·335416
	8¾	·236	·2364583′		8¾	·286	·2864583′		8¾	·336	·3364583′
4	9	·237	·2375	5	9	·287	·2875	6	9	·337	·3375
	9¼	·238	·2385416′		9¼	·288	·2885416′		9¼	·338	·3385416′
	9½	·239	·239583′		9½	·289	·289583′		9½	·339	·339583′
	9¾	·240	·240625		9¾	·290	·290625		9¾	·340	·340625
4	10	·241	·2416′	5	10	·291	·2916′	6	10	·341	·3416′
	10¼	·242	·2427083′		10¼	·292	·2927083′		10¼	·342	·3427083′
	10½	·243	·24375		10½	·293	·29375		10½	·343	·34375
	10¾	·244	·2447916′		10¾	·294	·2947916′		10¾	·344	·3447916′
4	11	·245	·24583′	5	11	·295	·29583′	6	11	·345	·34583′
	11¼	·246	·246875		11¼	·296	·296875		11¼	·346	·346875
	11½	·247	·247916′		11½	·297	·297916′		11½	·347	·347916′
	11¾	·248	·2489583′		11¾	·298	·2989583′		11¾	·348	·3489583′
5	0	·250	·250	6	0	·300	·300	7	0	·350	·350
	0¼	·251	·2510416′		0¼	·301	·3010416′		0¼	·351	·3510416′
	0½	·252	·252083′		0½	·302	·302083′		0½	·352	·352083′
	0¾	·253	·253125		0¾	·303	·303125		0¾	·353	·353125
5	1	·254	·25416′	6	1	·304	·30416′	7	1	·354	·35416′
	1¼	·255	·2552083′		1¼	·305	·3052083		1¼	·355	·3552083′
	1½	·256	·25625		1½	·306	·30625		1½	·356	·35625
	1¾	·257	·2572916′		1¾	·307	·3072916′		1¾	·357	·3572916′
5	2	·258	·2583′	6	2	·308	·3083′	7	2	·358	·3583′
	2¼	·259	·259375		2¼	·309	·309375		2¼	·359	·359375
	2 .	·260	·260416′		2½	·310	·310416′		2½	·360	·360416′
	2¾	·261	·2614582′		2¾	·311	·3114583′		2¾	·361	·3614583′
5	3	·262	·2625	6	3	·312	·3125	7	3	·362	·3625
	3¼	·263	·2635416′		3¼	·313	·3135416′		3¼	·363	·3635416′
	3½	·264	·264583′		3½	·314	·314583′		3½	·364	·364583′
	3¾	·265	·265625		3¾	·315	·315625		3¾	·365	·365625
5	4	·266	·266′	6	4	·316	·316′	7	4	·366	·36′
	4¼	·267	·2677083′		4¼	·317	·3177083′		4¼	·367	·3677083′
	4½	·268	·26875		4½	·318	·31875		4½	·368	·36875
	4¾	·269	·2697916′		4¾	·319	·3197916′		4¾	·369	·3697916′

s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.
7	5	·370	·37083′	8	5	·420	·42083′	9	5	·470	·47083′
	5¼	·371	·371875		5¼	·421	·421875		5¼	·471	·471875
	5½	·372	·372916′		5½	·422	·422916′		5½	·472	·472916′
	5¾	·373	·3739583′		5¾	·423	·4239583′		5¾	·473	·4739583′
7	6	·375	·375	8	6	·425	·425	9	6	·475	·475
	6¼	·376	·3760416′		6¼	·426	·4260416′		6¼	·476	·4760416′
	6½	·377	·377083′		6½	·427	·427083′		6½	·477	·477083′
	6¾	·378	·378125		6¾	·428	·428125		6¾	·478	·478125
7	7	·379	·37916′	8	7	·429	·42916′	9	7	·479	·47916′
	7¼	·380	·3802083′		7¼	·430	·4302083′		7¼	·480	·4802083′
	7½	·381	·38125		7½	·431	·43125		7½	·481	·48125
	7¾	·382	·3822916′		7¾	·432	·4322916′		7¾	·482	·4822916′
7	8	·383	·383′	8	8	·433	·43′	9	8	·483	·483′
	8¼	·384	·384375		8¼	·434	·434375		8¼	·484	·484375
	8½	·385	·385416′		8½	·435	·435416′		8½	·485	·485416′
	8¾	·386	·3864583′		8¾	·436	·4364583′		8¾	·486	·4864583′
7	9	·387	·3875	8	9	·437	·4375	9	9	·487	·4875
	9¼	·388	·3885416′		9¼	·438	·4385416′		9¼	·488	·4885416′
	9½	·389	·389583′		9½	·439	·439583′		9½	·489	·489583′
	9¾	·390	·390625		9¾	·440	·440625		9¾	·490	·490625
7	10	·391	·3916′	8	10	·441	·4416′	9	10	·491	·4916′
	10¼	·392	·3927083′		10¼	·442	·4427083′		10¼	·492	·4927083′
	10½	·393	·39375		10½	·443	·44375		10½	·493	·49375
	10¾	·394	·3947916′		10¾	·444	·4447916′		10¾	·494	·4947916′
7	11	·395	·39583′	8	11	·445	·44583′	9	11	·495	·49583′
	11¼	·396	·396875		11¼	·446	·446875		11¼	·496	·496875
	11½	·397	·397916′		11½	·447	·447916′		11½	·497	·497916′
	11¾	·398	·3989583′		11¾	·448	·4489583′		11¾	·498	·4989583′
8	0	·400	·400	9	0	·450	·450	10	0	·500	·500
	0¼	·401	·4010416′		0¼	·451	·4510416′		0¼	·501	·5010416′
	0½	·402	·402083′		0½	·452	·452083′		0½	·502	·502083′
	0¾	·403	·403125		0¾	·453	·453125		0¾	·503	·503125
8	1	·404	·40416′	9	1	·454	·45416′	10	1	·504	·50416′
	1¼	·405	·4052083′		1¼	·455	·4552083′		1¼	·505	·5052083′
	1½	·406	·40625		1½	·456	·45625		1½	·506	·50625
	1¾	·407	·4072916′		1¾	·457	·4572916′		1¾	·507	·5072916′
8	2	·408	·4083′	9	2	·458	·4583′	10	2	·508	·5083′
	2¼	·409	·409375		2¼	·459	·459375		2¼	·509	·509375
	2½	·410	·410416′		2½	·460	·460416′		2½	·510	·510416′
	2¾	·411	·4114583′		2¾	·461	·4614583′		2¾	·511	·5114583′
8	3	·412	·4125	9	3	·462	·4625	10	3	·512	·5125
	3¼	·413	·4135416′		3¼	·463	·4635416′		3¼	·513	·5135416′
	3½	·414	·414583′		3½	·464	·464583′		3½	·514	·514583′
	3¾	·415	·415625		3¾	·465	·465625		3¾	·515	·515625
8	4	·416	·416′	9	4	·466	·46′	10	4	·516	·516′
	4¼	·417	·4177083′		4¼	·467	·4677083′		4¼	·517	·5177083′
	4½	·418	·41875		4½	·468	·46875		4½	·518	·51875
	4¾	·419	·4197916′		4¾	·469	·4697916′		4¾	·519	·5197916′

s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.	s.	d.	Decimals of £1 in Mils.	Decimals of the present £1.
10	5	·520	·52083′	11	5	·570	·57083′	12	5	·620	·62083′
	5¼	·521	·521875		5¼	·571	·571875		5¼	·621	·621875
	5½	·522	·522916′		5½	·572	·572916′		5½	·622	·622916′
	5¾	·523	·5239583′		5¾	·573	·5739583′		5¾	·623	·6239583′
10	6	·525	·525	11	6	·575	·575	12	6	·625	·625
	6¼	·526	·5260416′		6¼	·576	·5760416′		6¼	·626	·6260416′
	6½	·527	·527083′		6½	·577	·577083′		6½	·627	·627083′
	6¾	·528	·528125		6¾	·578	·578125		6¾	·628	·628125
10	7	·529	·52916′	11	7	·579	·57916′	12	7	·629	·62916′
	7¼	·530	·5302083′		7¼	·580	·5802083′		7¼	·630	·6302083′
	7½	·531	·53125		7½	·581	·58125		7½	·631	·63125
	7¾	·532	·5322916′		7¾	·582	·5822916′		7¾	·632	·6322916′
10	8	·533	·53′	11	8	·583	·583′	12	8	·633	·63′
	8¼	·534	·534375		8¼	·584	·584375		8¼	·634	·634375
	8½	·535	·535416′		8½	·585	·585416′		8½	·635	·635416
	8¾	·536	·5364583′		8¾	·586	·5864583′		8¾	·636	·6364583′
10	9	·537	·5375	11	9	·587	·5875	12	9	·637	·6375
	9¼	·538	·5385416′		9¼	·588	·5885416′		9¼	·638	·6385416′
	9½	·539	·539583′		9½	·589	·589583′		9½	·639	·639583′
	9¾	·540	·540625		9¾	·590	·590625		9¾	·640	·640625
10	10	·541	·5416′	11	10	·591	·5916′	12	10	·641	·6416′
	10¼	·542	·5427083′		10¼	·592	·5927083′		10¼	·642	·6427083′
	10½	·543	·54375		10½	·593	·59375		10½	·643	·64375
	10¾	·544	·5447916′		10¾	·594	·5947916′		10¾	·644	·6447916′
10	11	·545	·54583′	11	11	·595	·59583′	12	11	·645	·64583′
	11¼	·546	·546875		11¼	·596	·596875		11¼	·646	·646875
	11½	·547	·547916′		11½	·597	·597916′		11½	·647	·647916′
	11¾	·548	·5489583′		11¾	·598	·5989583′		11¾	·648	·6489583′
11	0	·550	·5500	12	0	·600	·600	13	0	·650	·650
	0¼	·551	·5510416′		0¼	·601	·6010416′		0¼	·651	·6510416′
	0½	·552	·552083′		0½	·602	·602083′		0½	·652	·652083′
	0¾	·553	·553125		0¾	·603	·603125		0¾	·653	·653125
11	1	·554	·55416′	12	1	·604	·60416′	13	1	·654	·65416′
	1¼	·555	·5552083′		1¼	·605	·6052083		1¼	·655	·6552083′
	1½	·556	·55625		1½	·606	·60625		1½	·656	·65625
	1¾	·557	·5572916′		1¾	·607	·6072916′		1¾	·657	·6572916′
11	2	·558	·5583′	12	2	·608	·6083′	13	2	·658	·6583′
	2¼	·559	·559375		2¼	·609	·609375		2¼	·659	·659375
	2½	·560	·560416′		2½	·610	·610416′		2½	·660	·660416′
	2¾	·561	·5614583′		2¾	·611	·6114583′		2¾	·661	·6614583′
11	3	·562	·5625	12	3	·612	·6125	13	3	·662	·6625
	3¼	·563	·5635416′		3¼	·613	·6135416′		3¼	·663	·6635416′
	3½	·564	·564583′		3½	·614	·614583′		3½	·664	·664583′
	3¾	·565	·565625		3¾	·615	·615625		3¾	·665	·665625
11	4	·566	·56′	12	4	·616	·616′	13	4	·666	·6′
	4¼	·567	·5677083′		4¼	·617	·6177083′		4¼	·667	·6677083′
	4½	·568	·56875		4½	·618	·61875		4½	·668	·66875
	4¾	·569	·5697916′		4¾	·619	·6197916′		4¾	·669	·6697916′

S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.
13	5	·670	·67083′	14	5	·720	·72083′	15	5	·770	·77083′
	5¼	·671	·671875		5¼	·721	·721875		5¼	·771	·771875
	5½	·672	·672916′		5½	·722	·722916′		5½	·772	·772916′
	5¾	·673	·6739583′		5¾	·723	·7239583′		5¾	·773	·7739583′
13	6	·675	·675	14	6	·725	·725	15	6	·775	·775
	6¼	·676	·6760416′		6¼	·726	·7260416′		6¼	·776	·7760416′
	6½	·677	·677083′		6½	·727	·727083′		6½	·777	·777083′
	6¾	·678	·678125		6¾	·728	·728125		6¾	·778	·778125
13	7	·679	·67916′	14	7	·729	·72916′	15	7	·779	·77916′
	7¼	·680	·6802083′		7¼	·730	·7302083′		7¼	·780	·7802083′
	7½	·681	·68125		7½	·731	·73125		7½	·781	·78125
	7¾	·682	·6822916′		7¾	·732	·7322916′		7¾	·782	·7822916′
13	8	·683	·683′	14	8	·733	·73′	15	8	·783	·783′
	8¼	·684	·684375		8¼	·734	·734375		8¼	·784	·784375
	8½	·685	·685416′		8½	·735	·735416′		8½	·785	·785416′
	8¾	·686	·6864583′		8¾	·736	·7364583′		8¾	·786	·7864583′
13	9	·687	·6875	14	9	·737	·7375	15	9	·787	·7875
	9¼	·688	·6885416′		9¼	·738	·7385416′		9¼	·788	·7885416′
	9½	·689	·689583′		9½	·739	·739583′		9½	·789	·789583′
	9¾	·690	·690625		9¾	·740	·740625		9¾	·790	·790625
13	10	·691	·6916′	14	10	·741	·7416′	15	10	·791	·7916′
	10¼	·692	·6927083′		10¼	·742	·7427083′		10¼	·792	·7927083′
	10½	·693	·69375		10½	·743	·74375		10½	·793	·79375
	10¾	·694	·6947916′		10¾	·744	·7447916′		10¾	·794	·7947916′
13	11	·695	·69583′	14	11	·745	·74583′	15	11	·795	·79583′
	11¼	·696	·696875		11¼	·746	·746875		11¼	·796	·796875
	11½	·697	·697916′		11½	·747	·747916′		11½	·797	·797916′
	11¾	·698	·6989583′		11¾	·748	·7489583′		11¾	·798	·7989583′
14	0	·700	·700	15	0	·750	·750	16	0	·800	·800
	0¼	·701	·7010416′		0¼	·751	·7510416′		0¼	·801	·8010416′
	0½	·702	·702083′		0½	·752	·752083′		0½	·802	·802083′
	0¾	·703	·703125		0¾	·753	·753125		0¾	·803	·803125
14	1	·704	·70416′	15	1	·754	·75416′	16	1	·804	·80416′
	1¼	·705	·7052083′		1¼	·755	·7552083′		1¼	·805	·8052083′
	1½	·706	·70625		1½	·756	·75625		1½	·806	·80625
	1¾	·707	·7072916′		1¾	·757	·7572916′		1¾	·807	·8072916′
14	2	·708	·7083′	15	2	·758	·7583′	16	2	·808	·8083′
	2¼	·709	·709375		2¼	·759	·759375		2¼	·809	·809375
	2½	·710	·710416′		2½	·760	·760416′		2½	·810	·810416′
	2¾	·711	·7114583′		2¾	·761	·7614583′		2¾	·811	·8114583′
14	3	·712	·7125	15	3	·762	·7625	16	3	·812	·8125
	3¼	·713	·7135416′		3¼	·763	·7635416′		3¼	·813	·8135416′
	3½	·714	·714583′		3½	·764	·764583′		3½	·814	·814583′
	3¾	·715	·715625		¾	·765	·765625		3¾	·815	·815625
14	4	·716	·716′	15	4	·766	·76′	16	4	·816	·816′
	4¼	·717	·7177083′		4¼	·767	·7677083′		4¼	·817	·8177083′
	4½	·718	·71875		4½	·768	·76875		4½	·818	·81875
	4¾	·719	·7197916′		4¾	·769	·7697916′		4¾	·819	·8197916′

S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.
16	5	·820	·82083′	17	5	·870	·87083′	18	5	·920	·92083′
	5¼	·821	·821875		5¼	·871	·871875		5¼	·921	·921875
	5½	·822	·822916′		5½	·872	·872916′		5½	·922	·922916′
	5¾	·823	·8239583′		5¾	·873	·8739583′		5¾	·923	·9239583′
16	6	·825	·825	17	6	·875	·875	18	6	·925	·925
	6¼	·826	·8260416′		6¼	·876	·8760416′		6¼	·926	·9260416′
	6½	·827	·827083′		6½	·877	·877083′		6½	·927	·927083′
	6¾	·828	·828125		6¾	·878	·878125		6¾	·928	·928125
16	7	·829	·82916′	17	7	·879	·87916′	18	7	·929	·92916′
	7¼	·830	·8302083′		7¼	·880	·8802083′		7¼	·930	·9302083′
	7½	·831	·83125		7½	·881	·88125		7½	·931	·93125
	7¾	·832	·8322916′		7¾	·882	·8822916′		7¾	·932	·9322916′
16	8	·833	·83′	17	8	·883	·883′	18	8	·933	·93′
	8¼	·834	·834375		8¼	·884	·884375		8¼	·934	·934375
	8½	·835	·835416′		8½	·885	·885416′		8½	·935	·935416′
	8¾	·836	·8364583′		8¾	·886	·8864583′		8¾	·936	·9364583′
16	9	·837	·8375	17	9	·887	·8875	18	9	·937	·9375
	9¼	·838	·8385416′		9¼	·888	·8885416′		9¼	·938	·9385416′
	9½	·839	·839583′		9½	·889	·889583′		9½	·939	·939583′
	9¾	·840	·840625		9¾	·890	·890625		9¾	·940	·940625
16	10	·841	·8416′	17	10	·891	·8916′	18	10	·941	·9416′
	10¼	·842	·8427083′		10¼	·892	·8927083′		10¼	·942	·9427083′
	10½	·843	·84375		10½	·893	·89375		10½	·943	·94375
	10¾	·844	·8447916′		10¾	·894	·8947916′		10¾	·944	·9447916′
16	11	·845	·84583′	17	11	·895	·89583′	18	11	·945	·94583′
	11¼	·846	·846875		11¼	·896	·896875		11¼	·946	·946875
	11½	·847	·847916′		11½	·897	·897916′		11½	·947	·947916′
	11¾	·848	·8489583′		11¾	·898	·8989583′		11¾	·948	·9489583′
17	0	·850	·850	18	0	·900	·900	19	0	·950	·950
	0¼	·851	·8510416′		0¼	·901	·9010416′		0¼	·951	·9510416′
	0½	·852	·852083′		0½	·902	·902083′		0½	·952	·952083′
	0¾	·853	·853125		0¾	·903	·903125		0¾	·953	·953125
17	1	·854	·85416′	18	1	·904	·90416′	19	1	·954	·95416′
	1¼	·855	·8552083′		1¼	·905	·9052083′		1¼	·955	·9552083′
	1½	·856	·85625		1½	·906	·90625		1½	·956	·95625
	1¾	·857	·8572916′		1¾	·907	·9072916′		1¾	·957	·9572916′
17	2	·858	·8583′	18	2	·908	·9083′	19	2	·958	·9583′
	2¼	·859	859375		2¼	·909	·909375		2¼	·959	·959375
	2½	·860	860416′		2½	·910	·910416′		2½	·960	·960416′
	2¾	·861	·8614583′		2¾	·911	·9114583′		2¾	·961	·9614583′
17	3	·862	·8625	18	3	·912	·9125	19	3	·962	·9625
	3¼	·863	·8635416′		3¼	·913	·9135416′		3¼	·963	·9635416′
	3½	·864	·864583′		3½	·914	·914583′		3½	·964	·964583′
	3¾	·865	·865625		3¾	·915	·915625		3¾	·965	·965625
17	4	·866	86′	18	4	·916	·916′	19	4	·966	·96′
	4¼	·867	·8677083′		4¼	·917	·9177083′		4¼	·967	·9677083′
	4½	·868	·86875		4½	·918	·91875		4½	·968	·96875
	4¾	·869	·8697916′		4¾	·919	·9197916′		4¾	·969	·969716′

S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.	S.	D.	Decimals of £1 in Mils.	Decimals of the present £1.
19	5	·970	·97083′	19	7½	·981	·98125	19	10	·991	·9916′
	5¼	·971	·971875		7¾	·982	·9822916′		10¼	·992	·9927083′
	5½	·972	·972916′	19	8	·983	·983′		10½	·993	·99375
	5¾	·973	·9739583′		8¼	·984	·984375		10¾	·994	·9947916′
19	6	·975	·975		8½	·985	·985416′	19	11	·995	·99583′
	6¼	·976	·9760416′		8¾	·986	·9864583′		11¼	·996	·996875
	6½	·977	·977083′	19	9	·987	·9875		11½	·997	·997916′
	6¾	·978	·978125		9¼	·988	·9885416′		11¾	·998	·9989583′
19	7	·979	·97916′		9½	·989	·989583′				
	7¼	·980	·9802083′		9¾	·990	·990625				

TABLE II.

SHOWING THE VALUE OF FLORINS, CENTS, AND MILS, IN THE PRESENT DENOMINATIONS OF MONEY.

Florins, Cents, Mils.	S.	D.	Florins, Cents, Mils.	S.	D.	Florins, Cents, Mils.	S.	D.	Florins, Cents, Mils.	S.	D.
·001	0	$0\frac{6}{25}$	·020	0	$4\frac{4}{5}$	·039	0	$9\frac{9}{25}$	·110	2	$2\frac{2}{5}$
·002	0	$0\frac{12}{25}$	·021	0	$5\frac{1}{25}$	·040	0	$9\frac{3}{5}$	·120	2	$4\frac{4}{5}$
·003	0	$0\frac{18}{25}$	·022	0	$5\frac{7}{25}$	·041	0	$9\frac{21}{25}$	·125	2	6
·004	0	$0\frac{24}{25}$	·023	0	$5\frac{13}{25}$	·042	0	$10\frac{2}{25}$	·130	2	$7\frac{1}{5}$
·005	0	$1\frac{1}{5}$	·024	0	$5\frac{19}{25}$	·043	0	$10\frac{8}{25}$	·140	2	$9\frac{3}{5}$
·006	0	$1\frac{11}{25}$	·025	0	6	·044	0	$10\frac{14}{25}$	·150	3	0
·007	0	$1\frac{17}{25}$	·026	0	$6\frac{6}{25}$	·045	0	$10\frac{4}{5}$	·160	3	$2\frac{2}{5}$
·008	0	$1\frac{23}{25}$	·027	0	$6\frac{12}{25}$	·046	0	$11\frac{1}{25}$	·170	3	$4\frac{4}{5}$
·009	0	$2\frac{4}{25}$	·028	0	$6\frac{18}{25}$	·047	0	$11\frac{7}{25}$	·175	3	6
·010	0	$2\frac{2}{5}$	·029	0	$6\frac{24}{25}$	·048	0	$11\frac{13}{25}$	·180	3	$7\frac{1}{5}$
·011	0	$2\frac{16}{25}$	·030	0	$7\frac{1}{5}$	·049	0	$11\frac{19}{25}$	·190	3	$9\frac{3}{5}$
·012	0	$2\frac{22}{25}$	·031	0	$7\frac{11}{25}$	·050	1	0	·200	4	0
·013	0	$3\frac{3}{25}$	·032	0	$7\frac{17}{25}$	·060	1	$2\frac{2}{5}$	·210	4	$2\frac{2}{5}$
·014	0	$3\frac{9}{25}$	·033	0	$7\frac{23}{25}$	·070	1	$4\frac{4}{5}$	·220	4	$4\frac{4}{5}$
·015	0	$3\frac{3}{5}$	·034	0	$8\frac{4}{25}$	·075	1	6	·225	4	6
·016	0	$3\frac{21}{25}$	·035	0	$8\frac{2}{5}$	·080	1	$7\frac{1}{5}$	·230	4	$7\frac{1}{5}$
·017	0	$4\frac{2}{25}$	·036	0	$8\frac{16}{25}$	·090	1	$9\frac{3}{5}$	·240	4	$9\frac{3}{5}$
·018	0	$4\frac{8}{25}$	·037	0	$8\frac{22}{25}$	·100	2	0	·250	5	0
·019	0	$4\frac{14}{25}$	·038	0	$9\frac{3}{25}$						

TABLE III.

DAYS AS DECIMALS OF A YEAR.

Days.	Decimals of a Year.	Days.	Decimals of a Year.	Days.	Decimals of a Year.	Days.	Decimals of a Year.
1	·002739	43	·117808	85	·232876	127	·347945
2	·005479	44	·120547	86	·235616	128	·350684
3	·008219	45	·123287	87	·238356	129	·353424
4	·010958	46	·126027	88	·241095	130	·356164
5	·013698	47	·128767	89	·243835	131	·358904
6	·016438	48	·131506	90	·246575	132	·361643
7	·019178	49	·134246	91	·249315	133	·364383
8	·021917	50	·136986	92	·252054	134	·367123
9	·024657	51	·139726	93	·254794	135	·369863
10	·027397	52	·142465	94	·257534	136	·372602
11	·030136	53	·145205	95	·260273	137	·375342
12	·032876	54	·147945	96	·263013	138	·378082
13	·035616	55	·150684	97	·265753	139	·380821
14	·038356	56	·153424	98	·268493	140	·383561
15	·041095	57	·156164	99	·271232	141	·386301
16	·043835	58	·158904	100	·273972	142	·389041
17	·046574	59	·161643	101	·276712	143	·391780
18	·049314	60	·164383	102	·279452	144	·394520
19	·052054	61	·167123	103	·282191	145	·397260
20	·054794	62	·169863	104	·284931	146	·400000
21	·057534	63	·172602	105	·287671	147	·402739
22	·060273	64	·175342	106	·290410	148	·405479
23	·063013	65	·178082	107	·293150	149	·408219
24	·065753	66	·180821	108	·295890	150	·410956
25	·068493	67	·183561	109	·298630	151	·413698
26	·071232	68	·186301	110	·301369	152	·416438
27	·073973	69	·189041	111	·304109	153	·419178
28	·076712	70	·191780	112	·306849	154	·421917
29	·079452	71	·194520	113	·309589	155	·424657
30	·082191	72	·197260	114	·312328	156	·427397
31	·084931	73	·200000	115	·315068	157	·430136
32	·087671	74	·202739	116	·317808	158	·432876
33	·090410	75	·205479	117	·320547	159	·435616
34	·093150	76	·208219	118	·323287	160	·438356
35	·095890	77	·210958	119	·326027	161	·441095
36	·098630	78	·213698	120	·328767	162	·443835
37	·101360	79	·216438	121	·331506	163	·446575
38	·104109	80	·219178	122	·334246	164	·449315
39	·106849	81	·221917	123	·336986	165	·452054
40	·109589	82	·224657	124	·339726	166	·454794
41	·112328	83	·227397	125	·342465	167	·457534
42	·115068	84	·230136	126	·345205	168	·460273

Days.	Decimals of a Year.	Days.	Decimals of a Year.	Days.	Decimals of a Year.	Days.	Decimals of a Year.
169	·463013	218	·597260	267	·731506	316	·865753
170	·465753	219	·600000	268	·734246	317	·868493
171	·468493	220	·602739	269	·736986	318	·871232
172	·471232	221	·605479	270	·739725	319	·873972
173	·473972	222	·608219	271	·742465	320	·876712
174	·476712	223	·610958	272	·745205	321	·879452
175	·479452	224	·613698	273	·747945	322	·882191
176	·482191	225	·616438	274	·750685	323	·884931
177	·484931	226	·619178	275	·753424	324	·887671
178	·487671	227	·621917	276	·756164	325	·890410
179	·490410	228	·624657	277	·758904	326	·893150
180	·493150	229	·627397	278	·761644	327	·895890
181	·495890	230	·630136	279	·764383	328	·898630
182	·498630	231	·632876	280	·767123	329	·901369
183	·501369	232	·635616	281	·769863	330	·904109
184	·504109	233	·638356	282	·772603	331	·906849
185	·506849	234	·641095	283	·775343	332	·909589
186	·509589	235	·643835	284	·778083	333	·912328
187	·512328	236	·646575	285	·780822	334	·915068
188	·515068	237	·649315	286	·783562	335	·917808
189	·517808	238	·652054	287	·786302	336	·920547
190	·520547	239	·654794	288	·789042	337	·923287
191	·523287	240	·657534	289	·791781	338	·926027
192	·526027	241	·660273	290	·794520	339	·928767
193	·528767	242	·663013	291	·797260	340	·931506
194	·531506	243	·665753	292	·800000	341	·934246
195	·534246	244	·668493	293	·802739	342	·936986
196	·536986	245	·671232	294	·805479	343	·939726
197	·539726	246	·673972	295	·808219	344	·942465
198	·542465	247	·676712	296	·810958	345	·945205
199	·545205	248	·679452	297	·813698	346	·947945
200	·547945	249	·682191	298	·816438	347	·950684
201	·550684	250	·684931	299	·819178	348	·953424
202	·553424	251	·687671	300	·821917	349	·956164
203	·556164	252	·690410	301	·824657	350	·958904
204	·558904	253	·693150	302	·827397	351	·961643
205	·561643	254	·695890	303	·830136	352	·964383
206	·564383	255	·698630	304	·832876	353	·967123
207	·567123	256	·701369	305	·835616	354	·969863
208	·569863	257	·704109	306	·838356	355	·972602
209	·572602	258	·706849	307	·841095	356	·975342
210	·575342	259	·709589	308	·843835	357	·978082
211	·578082	260	·712328	309	·846575	358	·980821
212	·580821	261	·715068	310	·849315	359	·983561
213	·583561	262	·717808	311	·852054	360	·986301
214	·586301	263	·720547	312	·854794	361	·989041
215	·589041	264	·723287	313	·857534	362	·991780
216	·591780	265	·726027	314	·860273	363	·994520
217	·594520	266	·728767	315	·863013	364	·997260

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